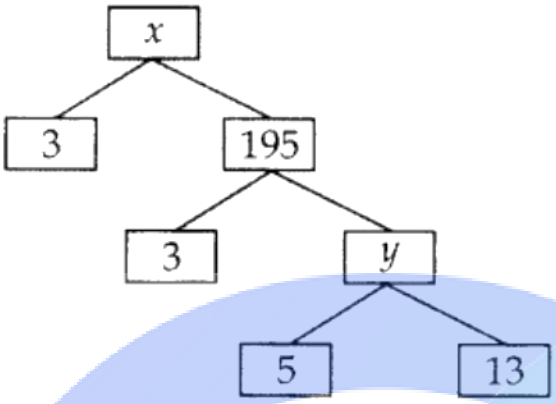
1. REAL NUMBERS

1. What is the least number that is divisible by all the numbers from 1 to 10?
2. Complete the following factor tree and find the composite number x and y .



3. Write the decimal representation of the rational number 8/27.
4. If HCF of a and b is 12 and product of these numbers is 1800. Then what is LCM of these numbers?
5. 5. If a is an odd number, b is not divisible by 3 and LCM of a and b is P, what is the LCM of 3a and 2b?
6. A rational number in its decimal expansion is 1.7351. What can you say about the prime factors of q when this number is expressed in the form p/q? Give reason.
7. Find LCM of numbers whose prime factorisation are expressible as 3×5^2 and $3^2 \times 7^2$.
8. Find the LCM of 96 and 360 by using fundamental theorem of arithmetic.
9. The LCM of two numbers is 14 times their HCF. The sum of LCM and HCF is 600. If one number is 280, then find the other number.
10. Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers. 198 and 144.
11. If two positive integers x and y are expressible in terms of primes as $x = p^2q^3$ and $y = p^3q$, what can you say about their LCM and HCF. Is LCM a multiple of HCF? Explain.
12. Find the largest number which divides 70 and 125 leaving remainder 5 and 8 respectively.
13. In a school, there are two Sections A and B of class X. There are 48 students in Section A and 60 students in Section B. Determine the least number of books required for the library of the school so that the books can be distributed equally among all students of each Section.
14. If n is an odd positive integer, show that $(n^2 - 1)$ is divisible by 8.
15. Prove that √7 is an irrational number
16. Find the largest number that will divide 398, 436 and 542 leaving remainders 7, 11, and 15 respectively.
17. Using prime factorisation method, find the HCF and LCM of 30, 72 and 432. Also show that HCF × LCM ≠ Product of the three numbers.
18. Amita, Sneha, and Raghav start preparing cards for all persons of an old age home. In order to complete one card, they take 10, 16 and 20 minutes respectively. If all of them started together, after what time will they start preparing a new card together?
19. Dudhnath has two vessels containing 720 ml and 405 ml of milk respectively. Milk from these containers is poured into glasses of equal capacity to their brim. Find the minimum number of glasses that can be filled.
20. There are 104 students in class X and 96 students in class IX in a school. In a house examination, the students are to be evenly seated in parallel rows such that no two adjacent rows are of the same class.

(a) Find the maximum number of parallel rows of each class for the seating arrangement.
(b) Also, find the number of students of class IX and also of class X in a row.
(c) What is the objective of the school administration behind such an arrangement?

Answer

1. Required number = LCM of 1, 2, 3, ... 10 = 2520.

2. $y = 5 \times 13 = 65$

a. $x = 3 \times 195 = 585$

3. Decimal representation of number 8/27 = 0.296

4. Product of two numbers = Product of their LCM and HCF

$\Rightarrow 1800 = 12 \times LCM$

$\Rightarrow LCM = 1800 / 12 = 150.$

5. 6P

6. As 1.7351 is a terminating decimal number, so q must be of the form $2^m 5^n$, where in, n are natural numbers.

7. LCM $(3 \times 5^2, 3^2 \times 7^2) = 3^2 \times 5^2 \times 7^2 = 9 \times 25 \times 49 = 11025$

8. $96 = 2^5 \times 3$

$360 = 2^3 \times 3^2 \times 5$

$LCM = 2^5 \times 3^2 \times 5 = 32 \times 9 \times 5 = 1440$

9. . ∴ LCM $(198,144) 2^4 \times 3^2 \times 11 = 1584$

HCF$(198, 144) = 2 \times 3^2 = 18$

Now, LCM (198, 144) × HCF (198, 144) = 1584 × 18 = 28512

and product of 198 and 144 = 28512

Thus, product of LCM (198, 144) and HCF (198, 144)

= Product of 198 and 144.

10. Let HCF of the numbers be x then according to question LCM of the number will be 14x

And $x + 14x = 600 \Rightarrow 15x = 600 \Rightarrow x = 40$

Then HCF = 40 and LCM = 14 × 40 = 560

∵ LCM × HCF = Product of the numbers

560 × 40 = 280 × Second number Second number = 560×40/280 = 80

Then other number is 80.

11. $x = p^2q^3$ and $y = p^3q$

$LCM = p^3q^3$

$HCF = p^2q$ .....(i)

Now, $LCM = p^3q^3$

$\Rightarrow LCM = pq^2 (p^2q)$

$\Rightarrow LCM = pq^2 (HCF)$

Yes, LCM is a multiple of HCF.

12. It is given that on dividing 70 by the required number, there is a remainder 5.

This means that 70 – 5 = 65 is exactly divisible by the required number.

Similarly, 125 – 8 = 117 is also exactly divisible by the required number.

$65 = 5 \times 13$

$117 = 3^2 \times 13$

HCF = 13

Required number = 13

13. Since the books are to be distributed equally among the students of Section A and Section B. therefore, the number of books must be a multiple of 48 as well as 60.

Hence, required number of books is the LCM of 48 and 60.

$48 = 2^4 \times 3$

$60 = 2^2 \times 3 \times 5$

LCM = $2^4 \times 3 \times 5 = 16 \times 15 = 240$

Hence, required number of books is 240.

14. If n is an odd positive integer, show that $(n^2 - 1)$ is divisible by 8.

We know that an odd positive integer n is of the form (4q + 1) or (4q + 3) for some integer q.

Case – I When n = (4q + 1)

In this case $n^2 - 1 = (4q + 1)^2 1 = 16q^2 + 8q = 8q(2q + 1)$

which is clearly divisible by 8.

Case – II When n = (4q + 3)

In this case, we have

$n2^2 = (4q + 3)^2 - 1 = 16q^2 + 24q + 8 = 8(2q^2 + 3q + 1)$

which is clearly divisible by 8.

Hence $(n^2 - 1)$ is divisible by 8.

15. Let us assume, to the contrary, that √7 is a rational number.

Then, there exist co-prime positive integers and such that

$\sqrt{7} = a/b$, $b \neq 0$

So, $a = \sqrt{7}\, b$

Squaring both sides, we have

$a^2 = 7b^2$ …… (i)

⇒ 7 divides a^2 ⇒ 7 divides a

So, we can write

a = 7c (where c is an integer)

Putting the value of a = 7c in (i), we have

$49c^2 = 7b^2\ 7^2 = b^2$

It means 7 divides b^2 and so 7 divides b.

So, 7 is a common factor of both a and b which is a contradiction.

So, our assumption that √7 is rational is wrong.

Hence, we conclude that √7 is an irrational number.

16. 398 – 7 = 391, 436 – 11 = 425, 542 – 15 = 527

HCF of 391, 425, 527 = 17

17. Given numbers = 30, 72, 432.

$30 = 2 \times 3 \times 5$; $72 = 2^3 \times 3^2$ and $432 = 2^4 \times 3^3$

So, HCF (30, 72, 432) = $2^1 \times 3^1 = 2 \times 3 = 6$

Again,

LCM (30, 72, 432) = $2^4 \times 3^3 \times 5^1 = 2160$

HCF × LCM = 6 × 2160 = 12960

Product of numbers = 30 × 72 × 432 = 933120 .

Therefore, HCF × LCM ≠ Product of the numbers.

18. To find the earliest (least) time, they will start preparing a new card together, we find the LCM of 10, 16 and 20.

$10 = 2 \times 5$

$16 = 2^4$

$20 = 2^2 \times 5$

LCM = $2^4 \times 5 = 16 \times 5 = 80$ minutes

They will start preparing a new card together after 80 minutes.

19. 1st vessel = 720 ml; 2nd vessel = 405 ml

We find the HCF of 720 and 405 to find the maximum quantity of milk to be filled in one glass.

$405 = 3^4 \times 5$

$720 = 2^4 \times 3^2 \times 5$

HCF = $3^2 \times 5 = 45$ ml = Capacity of glass

No. of glasses filled from 1st vessel = 720/45 = 16

No. of glasses filled from 2nd vessel = 405/45 = 9

Total number of glasses = 25

20. $104 = 2^3 \times 13$

$96 = 2^5 \times 3$

HCF = $2^3 = 8$

(a) Number of rows of students of class X = 104/8 = 13

Number maximum of rows class IX = 96/8 = 12

Total number of rows = 13 + 12 = 25

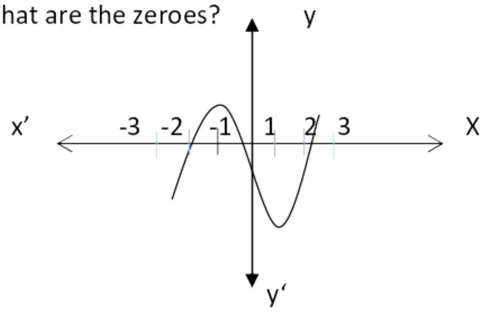
(b) No. of students of class IX in a row = 8

No. of students of class X in a row = 8

(c) The objective of school administration behind such an arrangement is fair and clean examination, so that no student can take help from any other student of his/her class.

2. POLYNOMIALS

1. From the given graph $y = p(x)$. Find the number of zeroes of the polynomial $p(x)$. What are the zeroes?



2. What will be the nature of the graph of the following polynomials

(i) $ax^2 + bx + c$ when $a > 0$

(ii) $ax^2 + bx + c$ when $a < 0$

3. What is the relation between a and b, if sum of the zeroes of the quadratic polynomial $ax^2 + bx + c$ $(a \neq 0)$ is equal to the product of the zeroes .

4. What is the degree of the polynomial whose graph intersect the x – axis at four points .

5. If -1 is one of the zeroes of the quadratic polynomial $ax^2 + bx + c$ $(a \neq 0)$, write at least one of its factor with justification.

6. If p and q are the zeroes of the quadratic polynomial $ax^2 + bx + c$ $(a \neq 0)$, find the value of $pq + (p+q)$

7. Find the zeroes of the polynomial $2x^2 - 3\sqrt{3}x + 3$

8. For what value of k, (-4) is a zero of the polynomial $x^2 - 2x - (3k + 3)$?

9. If α and β are the zeroes of the polynomial $x^2 - 4x - 12$, then find the value of $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$ without finding actual zeroes.

10. What should be subtracted from the polynomial of $p(x) = x^2 - 3ax + 3a - 7$ so that, $(x + 2)$ is a factor of the polynomial $p(x)$ and hence also find the value of a

11. If one of the zero of the polynomial $2x^2 - 4x - 2k$ is reciprocal of the other ,Find the value of k.

12. If α and β are the zeroes of the polynomial $2x^2 - 5x - 10$, then find the Value of $\alpha^{-2} + \beta^{-2}$ (by using algebraic identity)

13. Find the zeroes of the quadratic polynomial $2x^2 - 9 - 3x$ and verify the relationship between the zeroes and the coefficients

14. If two zeroes of the polynomial $x^3 - 4x^2 - 3x + 12$ are $\sqrt{3}$ and $-\sqrt{3}$, then find its third zero .

15. If α and β are the zeroes of the polynomial $f(x) = x^2 + px + q$ then form a polynomial whose zeroes are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

16. If the zeroes of the polynomial $x^2 + px + q$ are double in value to the zeroes of $2x^2 - 5x - 3$, Find the value of p and q.

17. If α and β are the zeroes of the polynomial $f(x) = x^2 - 9x + a$, find the value of a if $5\alpha + 4\beta = 40$

18. If -2 and the 3 are the zeroes of the polynomial $ax^2 + bx - 6$, then find the value of a and b

19. If the polynomial $f(x) = x^3 + 2x^2 - 5x + 1$ is divided by another polynomial $x + 3$, then the remainder comes out to be $ax + b$. Find the values of a and b (without doing actual division)

20. If 2 and -3 are the zeroes of the quadratic polynomial $x^2 + (a + 1)x + b$,Then find the value of a and b.

ANSWER

1. 3 zeroes, Zeroes are -2, 0, 2
2. (i) a parabola opening upward (ii) a parabola opening downward
3. $\frac{-b}{a} = \frac{c}{a}$ i.e $b + c = 0$
4. Degree is 4(since it has 4 zeroes)
5. By Factor theorem $(x+1)$ will be one of its factors if $x + 1 = 0 \Rightarrow x = -1$
6. $pq + (p+q) = \frac{c}{a} + \frac{-b}{a} = \frac{c-b}{a}$
7. $2x^2 - 3\sqrt{3}x + 3 = (2x - \sqrt{3})(x - \sqrt{3})$

   Zeroes $x = \sqrt{3}/2$ and $x = \sqrt{3}$
8. $p(x) = x^2 - 2x - (3k+3)$

   $\therefore p(-4) = (-4)^2 - 2(-4) - (3k+3)$

   $\Rightarrow 0 = 21 - 3k$ So, $k = 7$
9. $p(x) = x^2 - 4x - 12$

   $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta = \frac{\beta+\alpha}{\alpha\beta} - 2\alpha\beta = \frac{4}{-12} - 2(-12) = \frac{-4}{12} + 24 = \frac{-4+288}{12} = \frac{284}{12} = \frac{142}{6}$
10. If the polynomial $p(x) = x^2 - 3ax + 3a - 7$ is divided by $x + 2$, then by Remainder theorem, remainder is p(-2).

    $P(-2) = 9a - 3$

    $\therefore$ $9a - 3$ should be subtracted

    If $x + 2$ is a factor of $p(x)$, then by Factor theorem $p(-2) = 0$

    $\Rightarrow 9a - 3 = 0$

    So, $a = \frac{1}{3}$
11. Let α and $\frac{1}{\alpha}$ be the zeroes of the polynomial $p(x) = 2x^2 - 4x - 2k$

    $\therefore \alpha \times \frac{1}{\alpha} = \frac{-2k}{2}$ so, $k = -1$
12. $p(x) = 2x^2 - 5x - 10$

    $\alpha^{-2} + \beta^{-2} = \frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\beta^2 + \alpha^2}{(\alpha\beta)^2} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2} = \frac{\left(\frac{5}{2}\right)^2 - 2\left(-\frac{10}{2}\right)}{\left(-\frac{10}{2}\right)^2}$

    $\frac{\frac{25}{4} + 10}{\frac{100}{4}} = \frac{\frac{65}{4}}{\frac{100}{4}} = \frac{13}{20}$
13. $p(x) = 2x^2 - 3x - 9 = (2x + 3)(x - 3)$ [factorising by splitting of the middle term]

    Now, $p(x) = 0$ so, $x = \frac{-3}{2}$ and 3

    Sum of the zeroes $= \frac{-3}{2} + 3 = \frac{3}{2} = \frac{-\,coefficient\ of\ x}{coefficient\ of\ x^2}$

    Product if zeroes $= \frac{-3}{2} \times 3 = \frac{-9}{2} = \frac{constant\ tterm}{coefficient\ of\ x^2}$
14. As $\sqrt{3}$ and $-\sqrt{3}$ are the zeroes of the polynomial $x^3 - 4x - 3x + 12$,

    The quadratic polynomial forming by the given zeroes $= (x - \sqrt{3})(x + \sqrt{3})$

    $= x^2 - 3$

    Now, $x^3 - 4x - 3x + 12 = x^2(x - 4) - 3(x - 4) = (x^2 - 3)(x - 4)$

    $= (x - \sqrt{3})(x + \sqrt{3})(x - 4)$

    So the third zero of the given polynomial is 4
15. α and β are the zeroes of the polynomial $f(x) = x^2 + px + q$

    $\alpha + \beta = \frac{-p}{1} = -p$ and $\alpha \times \beta = \frac{q}{1} = q$

    so, $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta+\alpha}{\alpha\beta} = \frac{-p}{q}$ and $\frac{1}{\alpha} \times \frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{1}{q}$

The required polynomial is $p(x)$ = k [x^2 – (sum of zeroes)x + product of zeroes]

$= k\ [x^2 + \frac{p}{q}x + \frac{1}{q}]$

Taking $k = q$ $p(x) = qx^2 + px + 1$

16. The zeroes of the polynomial $2x^2 - 5x - 3$ are given by

$2x^2 - 5x - 3 = 0$

$(2x + 1)(x - 3) = 0$ so, $x = 3$ and $\frac{-1}{2}$

The zeroes of the polynomial $x^2 + px + q$ are 6 and – 1

Sum of the zeroes = 6+ (-1)

$-p = 5$ $\therefore$ $p = -5$

Product of the zeroes = 6× (-1)

$\therefore$ $q = -6$

17. α and β are the zeroes of the polynomial $f(x) = x^2 - 9x + a$

$\alpha + \beta = \frac{-(-9)}{1} = 9$

$5\alpha + 4\beta = 40$ $\Rightarrow \alpha + 4\alpha + 4\beta = 40$

$\Rightarrow \alpha + 4(\alpha + \beta) = 40$

$\Rightarrow \alpha + 4\times 9 = 40$ $\Rightarrow \alpha = 4$

Putting the value of α in $5\alpha + 4\beta = 40$ we get $\beta = 5$

So, product of the zeroes $\alpha\times\beta = \frac{a}{1}$ $\Rightarrow a = 4\times 5 = 20$

18. Let , $p(x) = ax^2 + bx - 6$

-2 and the 3 are the zeroes of the polynomial

Sum of the zeroes = - 2 + 3

$\Rightarrow \frac{-b}{a} = 1$ $\Rightarrow a = -b$…………………..(i)

Product of the zeroes = -2 × 3

$\Rightarrow \frac{c}{a} = -6$ $\Rightarrow \frac{-6}{a} = -6$ so , $a = 1$

From (i) we have $b = -1$

19. If $f(x) = x^3 + 2x^2 - 5x + 1$ is divided by another polynomial $x + 3$

So , by remainder theorem , remainder is $f(-3)$

Now, $f(-3) = (-3)^3 + 2(-3)^2 - 5(-3) + 1$

$= -27 + 18 + 15 + 1$

$= -27 + 34$

Remainder = 7 ………………………… (i)

But , remainder = $ax + b$ (given) ………… (ii)

Comparing (i) and (ii) we have ,

$ax + b = 0.x + 7$ so, $a = 0$ and $b = 7$

20. Let , $p(x) = x^2 + (a+1)x + b$

2 and – 3 are the zeroes of $p(x)$

so , $p(2) = 0$

$\Rightarrow 2^2 + (a+1)\times 2 + b = 0$

$\Rightarrow 2a + b = -6$ …………………… (i)

and $p(-3) = 0$

$\Rightarrow (-3)^2 + (a+1)\times(-3) + b = 0$

$\Rightarrow -3a + b = -6$ …………………… (i)

Solving equation (i) and (ii) we have $a = 0$ and $b = -6$

3. PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

1. A boat covers 14 kms in upstream and 20 kms downstream in 7 hours. Also it covers 22 kms upstream and 34 kms downstream in 10 hours. Find the speed of the boat in still water and of that the stream.

2. 6 men and 10 women can finish making pots in 8 days, while the 4 men and 6 women can finish it in 12 days. Find the time taken by one man alone and that of one woman alone to finish the work.

3. Draw the graph of 2x+y=6 and 2x-y+2=0. Shade the region bounded by these lines and x axis. Find the area of the shaded region?

4. A two-digit number is 4 more than 6 times the sum of its digit. If 18 is subtracted from the 21 number, the digits are reversed. Find the number.

5. A 2 digit number say z is exactly the four times the sum of its digits and twice the product of the digits. Find the number.

6. The sum of the ages of two children is 'a'. The age of the father is twice the 'a'. After twenty years, his age will be equal to the addition of the ages of his children. Find the age of father.

7. Find the value of p and q for which the system of equations represent coincident lines
   2x+3y=7,
   (p+q+1)x+(p+2q+2)y=4(p+q)+1

8. The length of the sides of a triangle are:
   $2x + \frac{y}{2}$,
   $\frac{5}{3}x + y + \frac{1}{2}$,
   $\frac{2}{3}x + 2y + \frac{5}{2}$
   If the triangle is equilateral, Find its perimeter?

9. Find graphically the vertices of triangle whose sides have the equations 2y-x=8, 5y-x=14, y-2x=1.

10. The ratio of two numbers is 2:3. If two is subtracted from the first number and 8 from the second, the ratio becomes the reciprocal of the original ratio. Find the numbers.

11. Solve the following system of equations:

$$\frac{x+y-8}{2} = \frac{x+2y-}{8} = \frac{3x+y-12}{11}$$

12. Rs. 4900 were divided among 150 children. If each girl gets Rs. 50 and a boy gets Rs. 25, then find the number of boys.

13. Solve for x and y
   $\frac{b}{a}x + \frac{a}{b}y = a^2 + b^2$; x+y = 2ab

14. If twice the son's age in years is added to the mother's age, the sum is 70 years. But if twice the mother's age is added to the son's age, the sum is 95 years. Find the age of the mother and her son.

15. The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 2 units the area increases by 67 square units. Find the dimensions of the rectangle.

16. Find the value of α and β for which the following pair of linear equation has infinite number of solutions.
$2x + 3y = 7$
$2 \alpha x + (\alpha + \beta) y = 28$

17. If 51x+23y=116 and 23x+51y=106, then find the value of (x-y).

18. One says, "Give me a hundred, friend! I shall then become twice as rich as you". The other replies, "If you give me ten, I shall be six times as rich as you". Tell me what is the amount of their (respective) capital?

19. Find the four angles of a cyclic quadrilateral ABCD in which $\angle A = (2x - 1)^\circ$, $\angle B = (y + 5)^\circ$, $\angle C = (2y + 15)^\circ$ and $\angle D = (4x - 7)^\circ$.

20. Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours if they travel towards each other they meet in 1 hour. What are the speeds of the two cars?

SOLUTIONS (Hints)

1. Let speed of boat in still water be x km/h and Speed of stream be y km/h
Speed upstream = (x + y) km/h
Speed downstream = (x – y) km/h

   Using formula, T = D/S, write equations and solve them.

2. Let time taken by one man alone to complete the work be x days
And time taken by one woman alone to complete the work be y days

   Find the work done by 1 man and by 1 woman in one day. Form the equation according to one day's work and solve them.
3. Form tables and find different values of x and y for both the equations. Plot the point and draw lines of both the equations. Shade the triangle and find its area.
Area of triangle = $\frac{1}{2}$ x b x h

4. Let the unit's place digit of the number be x and the ten's place digit be y.
   Number = 10y + x
   ATQ
   10y + x = 4 + 6 (y +x)

   10y + x - 18 = 10x + y

   Solve for x and y and find the number.
5. z = 10a+b
   ATQ
   10a+b=4(a+b)
   10a+b=2ab
   Equate and solve for a and b and find z.
6. Let the sum of the ages of two children will be a and age of father will be 2a.
   A.T.Q.,
   a+40(as 20 years is added in age of both children) = 2a+20
   Solve for a and find father's age.

7. For coincident lines,
   $\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$
   Write the equations in standard form and put values of a1, a2,b1,b2,c1 and c2 and find values of p and q.

8. In an Equilateral triangle, all sides are equal.
   Hence, equate all the three equations and solve for x and y.
   Find the sides and then the perimeter of the triangle.

9. Plot the graph of all the three given equations. And mark the intersecting point.
   Intersecting points will be the vertices of the triangle.

10. Let the numbers be x and y.
    ATQ,
    $\frac{x}{y} = \frac{2}{3}$
    Also,
    $\frac{x-2}{y-8} = \frac{3}{2}$
    Solve for x and y
11. First equate $\frac{x+y-8}{2}$ and $\frac{x+2y-}{8}$
    Then equate $\frac{x+2y-14}{8}$ and $\frac{3x+y-}{11}$
    and solve for x and y

12. Let number of boys be x and number of girls be y.
    ATQ
    x + y = 150

25x + 50y = 4900
Solve for x.

13. Solve for x and y (Solution will be in form of a and b)

14. Let Son's age be x years and Mother's age be y years
    $2x + y = 70$
    $2y + x = 95$
    Solve for x and y

15. Let the length of the rectangle be x units and breadth be y units.
    ATQ
    $(x - 5)(y + 3) = xy - 9$
    $(x + 3)(y + 2) = xy + 67$
    Solve for x and y.

16. Write the condition for infinite solutions. Put values of a1,a2, b1,b2, c1 and c2 and solve for α and β

17. Solve both the equations for x and y. Find x– y.

18. Let amount of money with first person = Rs. x
    Let amount of money with second person = Rs. Y
    ATQ
    $x + 100 = 2(y - 100)$
    Also,
    $y + 10 = 6(x - 10)$
    Solve for x and y.

19. For cyclic quadrilateral, opposite angles are supplementary. Using this theorem, form equations and find the angles.
20. Let the speed of Car starting from A be x km/h and speed of the car starting from B be y km/h.
    ATQ,
    $x - y = 100/5$
    $x + y = 100/1$
    Solve for x and y

4. QUADRATIC EQUATIONS

Q. Find the roots of the following quadratic equations by factorization:

1. $4x^2 - 4a^2x + (a^4 - b^4) = 0$

2. $\sqrt{\frac{x}{1-x}} + \sqrt{\frac{1-x}{x}} = 2\frac{1}{6}, \quad x \neq 0, 1$

3. $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$

4. $\frac{4x-3}{2x+1} - 10\frac{2x+1}{4x-3} = 3, \quad x \neq \frac{-1}{2}, \frac{3}{4}$

5. $(12abx^2) - (9a^2 - 8b^2)x - 6ab = 0$

6. If the roots of the quadratic equation: $(a^2 + b^2)x^2 + 2(bc - ad)x + (c^2 + d^2) = 0$ are real and equal, show that $ac + bd = 0$.

1. If the roots of the quadratic equation: $p(q-r)x^2 + q(r-p)x + r(p-q) = 0$ are equal, then show that $\frac{1}{p} + \frac{1}{r} = \frac{2}{q}$.

8 .If the equation $(1 + m^2)x^2 + 2mcx + (c^2 - a^2) = 0$ has equal roots, Prove that $c^2 = a^2(1 + m^2)$.

9. If the roots of the equation:

$(c^2 - ab)x^2 - 2(a^2 - cb)x + b^2 - ac = 0$ are equal, prove that either $a = 0$ or $a^3 + b^3 + c^3 = 3abc$

Q. Find the roots of the following quadratic equations, by applying the quadratic formula:

10. $9x^2 - 3(a+b)x + ab = 0$

11. $9x^2 - 3(a^2 + b^2)x + a^2b^2 = 0$

12. $a^2b^2x^2 - (4b^4 - 3a^4)x - 12a^2b^2 = 0$

13. $9x^2 - 9(a+b)x + (2a^2 + 5ab + 2b^2) = 0$

14. $x^2 - 2(a^2 + b^2)x + (a^2 - b^2)^2 = 0$

Q. Solve for x:

15. $(x^2 - 5x)^2 - (x^2 - 5x) + 6 = 0$

16. $9^{x+2} - 6.3^{x+1} + 1 = 0$

17. $4\left(x - \frac{1}{x}\right)^2 - 4\left(x + \frac{1}{x}\right) + 1 = 0$

18. $\left(x^2 + \frac{1}{x^2}\right) - 3\left(x - \frac{1}{x}\right) - 2 = 0$

19. $\left(\frac{2x}{x-5}\right)^2 + \left(\frac{10x}{x-5}\right) - 24 = 0, (x \neq 5)$

ANSWER

1. $\left(\frac{a^2+b^2}{2}, \frac{a^2-b^2}{2}\right)$
2. $\left(\frac{9}{13}, \frac{4}{13}\right)$
3. $(-a, -b)$
4. $\left(\frac{-4}{3}, \frac{1}{8}\right)$
5. $\left(\frac{3a}{4b}, -\frac{2b}{3a}\right)$

10. $\left(\frac{a}{3}, \frac{b}{3}\right)$

11. $\left(\frac{a^2}{3}, \frac{b^2}{3}\right)$

12. $\left(\frac{4b^2}{a^2}, \frac{-3a^2}{b^2}\right)$

13. $\left(\frac{2a+b}{3}, \frac{a+2b}{3}\right)$

14. $((a+b)^2, (a-b)^2)$

15. $(x = 6, -1, \frac{5+\sqrt{29}}{2}, \frac{5-\sqrt{29}}{2}$

16. (x = -2)

17. (x = 2, ½)

18. $\left(-1, 1, \frac{3 \pm \sqrt{13}}{2}\right)$

19. (x = 4, 15]

5.ARITHMETIC PROGRESSIONS

1. If a, (a - 2) and 3a are in AP, then the Value of a is:
(a) -3
(b) -2
(c) 3
(d) 2

2. What is the common difference of AP in which $a_{21} - a_7 = 84$?

3. Calculate the common difference of AP: $\frac{1}{2b}, \frac{1-6b}{2b}, \frac{1-12b}{2b}, \ldots \ldots$

4. Which is the first negative term of the AP: 35, 30, 25, 20...?
(a) 7th Term
(b) 5th Term
(c) 9th Term
(d) 11th Term

5. Find the next term of the arithmetic progression: $\sqrt{12}, \sqrt{27}, \sqrt{48}$,......
(a) $\sqrt{75}$
(b) $\sqrt{60}$
(c) $\sqrt{80}$
(d) $\sqrt{90}$

6. For what value of 'k' will k + 9, 2k – 1 & 2k + 7 are the consecutive terms of an AP?

7. How many terms of the AP 27, 24, 21, ... should be taken so that their sum is zero?

8. Find the sum of first 8 multiples of 3.

9. In an AP, if $S_5 + S_7 = 167$ & $S_{10} = 235$, then find the AP, where S_n denotes the sum of its first 'n' terms.

10. The first & the last terms of an AP are 7 & 49 respectively. If sum of all its terms is 420, find its common difference.

11. Find the number of natural numbers between 101 & 999 which are divisible by both 2 & 5.

12. Find the sum of n terms of the series $\left(4-\frac{1}{n}\right)+\left(4-\frac{2}{n}\right)+\left(4-\frac{3}{n}\right)+\ldots\ldots\ldots$

13. If the sum of the first n terms of an AP is $\frac{1}{2}(3n^2+7n)$, then find its nth term. Hence write its 20th term.

14. If Sn denotes the sum of first n terms of an AP. Prove that S12 = 3(S8 – S4).

15. If the sum of first p terms of an A.P. is the same as the first q terms (where p ≠ q), then show that the sum of first (p + q) terms is zero.

16. The ratio of the sums of first m & first n terms of an AP is $m^2 : n^2$. Show that the ratio of its mth & nth terms is (2m – 1) : (2n – 1).

17. The sum of three numbers in an AP is 12 & sum of their cubes is 288. Find the numbers.

18. The sum of four consecutive numbers in an AP. Is 32 & the ratio of the product of the first & the last term to the product of two middle terms is 7:15. Find the numbers.

HINTS

1. Since a, a-2 and 3a are in AP
$\therefore a-2-a=3a-(a-2)$
$\Rightarrow 2(a-2)=a+3a$
$\Rightarrow 2a-4=4a$
$\Rightarrow 2a=-4$
$\Rightarrow a=-2$

2. 2. Let the common difference of an A.P. be d.

Then,

a18=a1+17×d

a14=a1+13×d

Solving the two equations,

a18−a14=a1+17d−a1−13d

⇒a18−a14=4d

Substituting 4d=32,

⇒d=8

3. D=a_2-a_1

ie,d=1-6b/2b - 1/2b

1-6b-1/2b

-6b/2b

-3 is the answer

4. Here, a = 18, d = $-\frac{5}{2}$

a_n = a + (n - 1) d

=> - 47 = 18 + (n - 1) $-\frac{5}{2}$

=>5n/2 = 18 + 47 + 5/2 = 67.5

Hence, it is 27th term

5. write √12 as 2√3, √27 as 3√3, √48 as 4√3.

So this forms a AP with common difference =√3

next term will be 5√3=75

6. Let,

k + 9 = a

2k - 1 = b

2k + 7 = c

To be in AP,

a + c = 2b

(k + 9) + (2k + 7) = 2(2k - 1)

k + 9 + 2k + 7 = 4k - 2

3k + 16 = 4k - 2

3k - 4k = - 2 - 16

- k = - 18

k = 18

For k = 18, the terms k+9 , 2k - 1 , 2k + 7 are in AP

7. Let first term be a=27

And common difference be d=−3

According to question, sum is zero,

⟹n/2[2a+(n−1)d]=0

⟹[54+(n−1)(−3)]=0

⟹n=19

Hence, 19 terms of AP should be taken to make sum zero.

8. First 8 multiples of 3-
3,6,9,.....upto 8 terms
The above series is in A.P. where,
First term (a)=3
Common difference (d)=3
No. of terms (n)=8
Sum of terms (Sn)=?
As we know that, in an A.P.,
$S_n=n/2[2a+(n-1)d]$
$\therefore S_8=8/2[2\times3+(8-1)\times3]$
$\Rightarrow S_8=4\times(6+21)$
$\Rightarrow S_8=4\times27=108$

9. Let the first term is a and the common difference is d
By using $S_n=n/2[2a+(n-1)d]$ we have,
$S_5=5/2[2a+(5-1)d]$
$=5/2[2a+4d]$
$S_7=7/2[2a+(7-1)d]=7/2[2a+6d]$
Given: $S_7+S_5=167$
$\therefore 5/2[2a+4d]+7/2[2a+6d]=167$

$\Rightarrow 10a+20d+14a+42d=334$
$\Rightarrow 24a+62d=334$ ...(1)
$S_{10}=10/2[2a+(10-1)d]=5(2a+9d)$
Given: $S_{10}=235$
So $5(2a+9d)=235$
$\Rightarrow 2a+9d=47$ ...(2)
Multiply equation (2) by 12, we get
$24a+108d=564$....(3)
Subtracting equation (3) from (1), we get
$-46d=-230$
$\therefore d=5$
Substing the value of d=5 in equation (1) we get
$2a+9(5)=47$ or $2a=2$
$\therefore a=1$
Then A.P is $1,6,11,16,21,\cdots$

10. a=7 l=49 Sn=420
$S_n=n/2[a+l]$
So $420\times2=n[7+49]$
n=15
$l=a+(n-1)d$
$\Rightarrow 49=7+14d$
$\Rightarrow 7=1+2d \Rightarrow 2d=6$
$\Rightarrow d=3$

11. The list of numbers between 101 and 999 that are divisible by 2 and 5 are:
110,120,130,...990

The numbers are in A.P, with first term, a=110, common difference, d=10
Last term, an=990
We know that, an=a+(n−1)d
990=110+(n−1)10
⇒990–110=10n–10
⇒880+10=10n
⇒890=10n
⇒n=89
Therefore, the number of terms between 101 and 999 that are divisible by 2 and 5 are 89.

12. (4 + 4 + 4 + 4 + 4 + ... upto n terms) + (-1/n - 2/n - 3/n - ....upto n terms)
= 4 (1+1+1+1.......... upto n terms) - 1/n (1 + 2 + 3 +4 ....upto n terms)

13. Sn=1/2(3n^2+7n)
S1=1/2(3+7)=5
S2=1/2(3*4+7*2)=26/2=13
We know
S1=a1=5
S2=a1+a2=13
S2-s1=a1+a2-a1
13-5=a2
a2=8
We know d=a2-a1
d=8-5=3
nth term of AP =an=5+(n-1)3
an= 2+3n
Therefore 20th term =
a20= 2+3(20)=62
Hence 20th term of AP is 62

14. let a is the first term of Ap and d is the common difference
Sn=n/2 {2a+(n-1) d

now S12=12/2 {2a+(12-1) d}=12a+66d
S8=8/2 {2a+7d}=8a+28d
S4=4/2 {2a+3d}=4a+6d
LHS=S12=12a+66d
RHS=3 (S8-S4)=3 (8a+28d-4a-6d)=12a+66d
LHS =RHS

15. Sp=Sq
⇒ p/2(2a+(p−1)d)=q/2(2a+(q−1)d)
⇒ p(2a+(p−1)d)=q(2a+(q−1)d)
⇒ $2ap+p^2d-pd=2aq+q^2d-qd$
⇒ 2a(p−q)+(p+q)(p−q)d−d(p−q)=0
⇒ (p−q)[2a+(p+q)d−d]=0
⇒ 2a+(p+q)d−d=0
⇒ 2a+((p+q)−1)d=0

⇒ Sp+q=0

16. (HINT)Let Sm and Sn be the sum of the first m and first n terms of the AP respectively. Let, a be the first term and d be a common difference

$Sn/Sm=n^2/m^2$

17. Hera 3a=12

a=4

Also $(a-d)^3+a^3+(a+d)^3=288$,

or $^{3a3}+6ad^2=288$

$24d^2=288-3\times64=96$

$d^2=4$

$d=\pm2$

Hence the numbers are 2,4,6 or 6,4,2

18. Let the four consecutive numbers in AP be (a−3d),(a−d),(a+d) and (a+3d)

So, according to the question.

a−3d+a−d+a+d+a+3d=32

4a=32

a=32/4

a=8......(1)

Now, (a−3d)(a+3d)/(a−d)(a+d)=7/15

$15(a^2-9d^2)=7(a^2-d^2)$

$15a^2-135d^2=7a^2-7d^2$

$15a^2-7a^2=135d^2-7d^2$

$8a^2=128d^2$

Putting the value of a=8 in above we get.

$8(8)^2=128d^2$

$128d^2=512$

$d^2=512/128$

$d^2=4$

d=2

So, the four consecutive numbers are

8−(3×2)

8−6=2

8−2=6

8+2=10

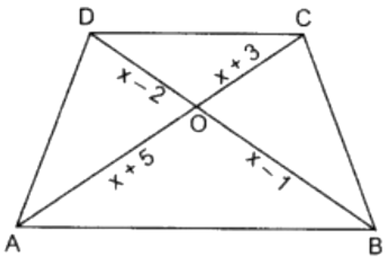
8+(3×2)

8+6=14

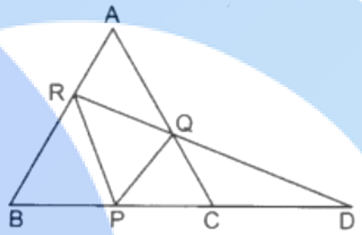
Four consecutive numbers are 2,6,10and14.

6. TRIANGLE

1. In the given figure, if AB || DC, find the value of x.

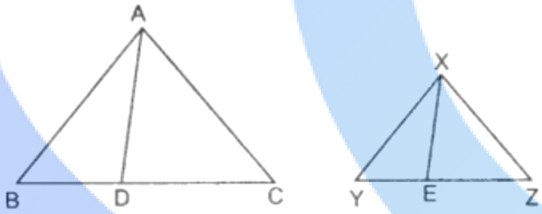


2. In the given figure PQ || BA; PR || CA. If PD = 12 cm. Find BD X CD.

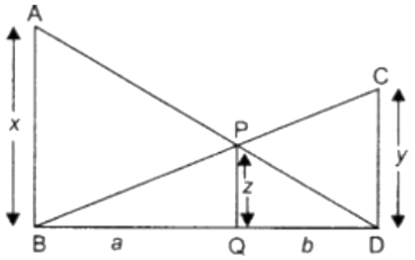


3. If one diagonal of a trapezium divides the other diagonal in the ratio 1 : 3. Prove that one of the parallel sides is three times the other.

4. In given figure ΔABC is similar to ΔXYZ and AD and XE are angle bisectors of ∠A and ∠X respectively such that AD and XE in cm are 4 and 3 respectively, find the ratio of area of ΔABD and area of ΔXYE.



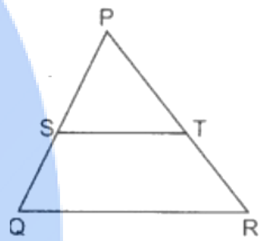
5. In figure, AB || PQ || CD, AB = x units, CD =y units and PQ = z units, prove that
$\frac{1}{x}+\frac{1}{y}=\frac{1}{z}$



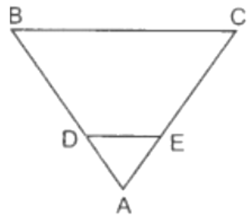
6. The area of two similar triangles are 49 cm^2 and 64 cm^2 respectively. If the difference of the corresponding altitudes is 10 cm, then find the lengths of altitudes (in centimetres).

7. In an equilateral triangle ABC, D is a point on side BC such that 4BD = BC. Prove that ADcm^2 = BCcm^2.

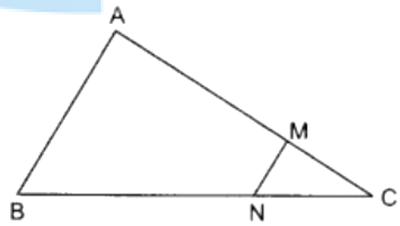
8. In figure, S and T are points on the sides PQ and PR, respectively of APQR, such that PT = 2 cm, TR = 4 cm and ST is parallel to QR. Find the ratio of the areas of ΔPST and ΔPQR.



9. In figure, DE || BC in AABC such that that BC = 8 cm, AB = 6 cm and DA = 1.5 cm. Find DE.



10. In figure, MN || AB, BC = 7.5 cm, AM = 4 cm and MC = 2 cm. Find the length BN.

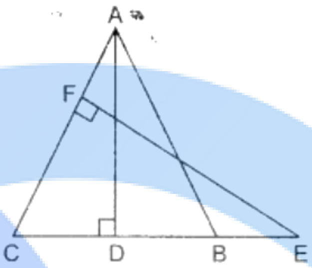


11. Triangle ABC is right angled at B, and D is mid-point of BC. Prove that $AC^2 = 4AD^2 - 3AB^2$.

12. If BL and CM are medians of a triangle ABC right angled at A, then prove that $4(BL^2 + CM^2) = 5BC^2$.

13. In figure, a triangle ABC is drawn to circumscribe a circle of radius 3 cm, such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 6 cm and 8 cm respectively. Find the side AB if the area of ΔABC = 63 cm2.

14. Prove that in a right angle triangle the square on the hypotenuse is equal to the sum of the squares on the other two sides. Point D is the mid-point of the side BC of a right triangle ABC, right angled at C. Prove that, $4AD^2 = 4AC^2 + BC^2$..



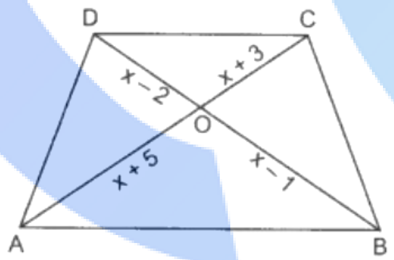
15. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.
Using the above, prove the following:
If the areas of two similar triangles are equal, then prove that the triangles are congruent.

ANSWER

1.

AB || DC ∴ ΔDOC ~ ΔBOA

$$\frac{OD}{OB} = \frac{OC}{OA} \Rightarrow \frac{x-2}{x-1} = \frac{x+3}{x+5}$$



2.

In ΔBRD,

$$BR \parallel PQ$$

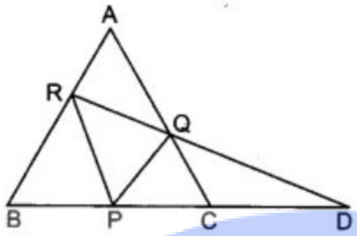
$$\therefore \quad \frac{BD}{PD} = \frac{RD}{QD}$$

In ΔRDP, $\quad PR \parallel QC$

$$\therefore \quad \frac{RD}{QD} = \frac{PD}{CD}$$

From (*i*) and (*ii*), we get

$$\frac{PD}{CD} = \frac{BD}{PD}$$



3.

$$DE : EB = 1 : 3$$

In ΔAEB and ΔCED, $\angle 1 = \angle 2$ (Alternate angles)

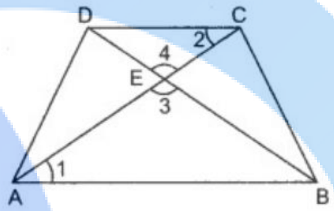
$$\angle 3 = \angle 4 \quad \text{(V.O.A.)}$$

$$\therefore \quad \Delta AEB \sim \Delta CED$$

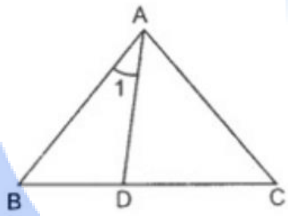
$$\Rightarrow \quad \frac{AB}{CD} = \frac{BE}{DE}$$

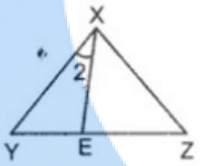
$$\Rightarrow \quad \frac{AB}{CD} = \frac{3}{1} \quad [\because DE : BE = 1 : 3]$$

$$\Rightarrow \quad AB = 3CD$$



4.





AD bisects $\angle A$ $\therefore$ $\angle 1 = \frac{1}{2}\angle A$

Similarly $\quad \angle 2 = \frac{1}{2}\angle X$

$$\because \quad \Delta ABC \sim \Delta XYZ$$

$$\therefore \quad \angle A = \angle X$$

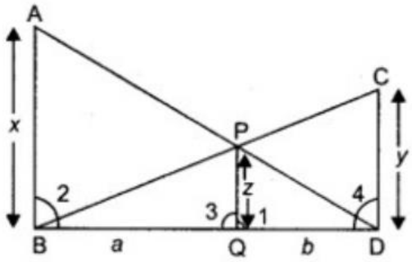
$$\Rightarrow \quad \frac{1}{2}\angle A = \frac{1}{2}\angle X \Rightarrow \angle 1 = \angle 2$$

Also $\quad \angle B = \angle Y$

$$\therefore \quad \Delta ABD \sim \Delta XYE$$

$$\frac{\text{Area } \Delta ABD}{\text{Area } \Delta XYE} = \frac{AD^2}{XE^2} = \frac{4^2}{3^2} = \frac{16}{9}$$

5.



In ΔADB and ΔPDQ,

$\because$ $PQ \parallel AB$ $\therefore \angle 1 = \angle 2$,

and $\angle ADB = \angle PDQ$

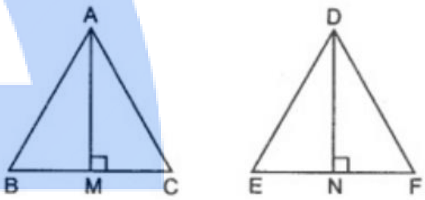
$\therefore$ $\Delta ADB \sim \Delta PDQ$

Similarily $\Delta BCD \sim \Delta BPQ$

$\because$ $\Delta ADB \sim \Delta PDQ$

$\therefore$ $\dfrac{AB}{PQ} = \dfrac{BD}{DQ}$

6.



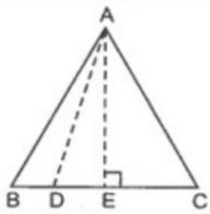
$\Delta ABC \sim \Delta DEF$

$\therefore$ $\dfrac{ar(\Delta ABC)}{ar(\Delta DEF)} = \dfrac{BC^2}{EF^2}$

$\Rightarrow$ $\dfrac{49}{64} = \dfrac{BC^2}{EF^2} \Rightarrow \dfrac{BC}{EF} = \dfrac{7}{8}$

Also $\dfrac{ar(\Delta ABC)}{ar(\Delta DEF)} = \dfrac{\frac{1}{2}BC \times AM}{\frac{1}{2}EF \times DN}$ $\Rightarrow$ $\dfrac{49}{64} = \dfrac{BC}{EF} \times \dfrac{AM}{DN}$

7.



Construction: Draw $AE \perp BC$.

$\therefore$ $BE = \frac{1}{2}BC$.

In right ΔAED,

$AD^2 = DE^2 + AE^2$

$\Rightarrow$ $AE^2 = AD^2 - DE^2$

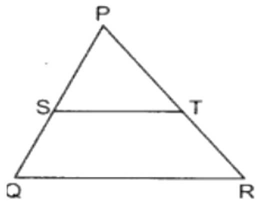
In right ΔAEB,

$AB^2 = AE^2 + BE^2$

$\Rightarrow$ $AB^2 = AD^2 - DE^2 + BE^2$

$\Rightarrow$ $AB^2 + DE^2 - BE^2 = AD^2$

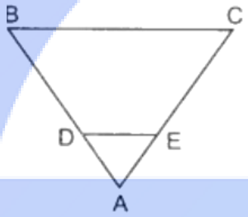
8.



In ΔPST and ΔPQR

$$\angle S = \angle Q$$
$$\angle P = \angle P$$
$$\therefore \quad \Delta PST \sim \Delta PQR$$
$$\therefore \quad \frac{ar(\Delta PST)}{ar(\Delta PQR)} = \frac{PT^2}{PR^2}$$

9.

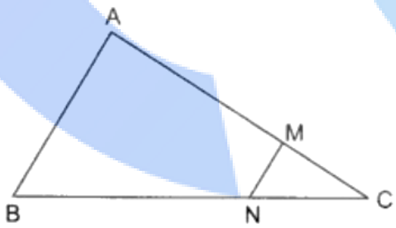


DE || BC

In ΔADE and ΔABC,

$$\angle ADE = \angle ABC$$
$$\angle A = \angle A$$
$$\therefore \quad \Delta ADE \sim \Delta ABC$$
$$\Rightarrow \quad \frac{AD}{AB} = \frac{DE}{BC}$$

10.

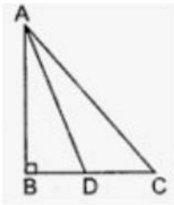


In ΔABC, MN || AB $\Rightarrow \Delta ABC \sim \Delta MNC$

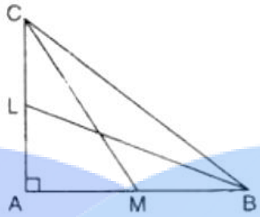
$$\Rightarrow \quad \frac{MC}{AM} = \frac{NC}{BN}$$

11.

Use Pythagoras Theorem



Use Pythagoras Theorem



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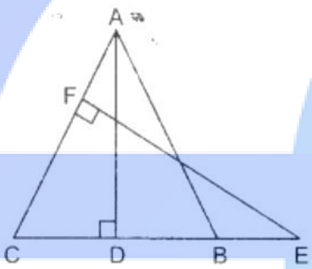
13.



In ΔADB and ΔEFC,

$\angle D = \angle F$ [Each 90°]

and $\angle B = \angle C$

[Angles opp. to equal sides of a triangle are equal]

$\Rightarrow$ $\Delta ABD \sim \Delta ECF$ [AA similarity]

$\therefore$ $\frac{AB}{EC} = \frac{AD}{EF}$ [Corresponding

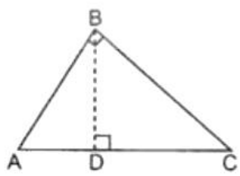
$\therefore$ $AB \times EF = AD \times EC$

14.

Construction: Draw $BD \perp AC$

Hints: Since, in ΔABC, $\angle B = 90°$ and $BD \perp AC$

so, $\Delta ADB \sim \Delta ABC$ [If a $\perp$ is drawn from the vertex of the rt. angle of rt. Δ to the hypotenuse then Δ's on both sides of the $\perp$ are similar to the whole Δ and to each other]



15.

To prove: $\Delta ABC \cong \Delta PQR$

Proof: Using the above result, we have

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{AC^2}{PR^2} = \frac{BC^2}{QR^2}$$

Also $ar(\Delta ABC) = ar(\Delta PQR)$ [Given]

$\therefore$ $1 = \frac{AB^2}{PQ^2} = \frac{AC^2}{PR^2} = \frac{BC^2}{QR^2}$

$\Rightarrow$ $AB = PQ, AC = PR, BC = QR$

$\Rightarrow$ $\Delta ABC \cong \Delta PQR$ [SSS]

7. COORDINATE GEOMETRY

1. Find the distance between the following pair of points:
   (a + b, b + c) and (a – b, c – b)
2. Find the value of a when the distance between the points (3, a) and (4, 1) is √10
3. If the points (2, 1) and (1, -2) are equidistant from the point (x, y), show that x + 3y =0.
4. 4. Find the values of x, y if the distances of the point (x, y) from (-3, 0) as well as from (3, 0) are 4.
5. Show that the points (-4, -1), (-2, -4), (4, 0) and (2, 3) are the vertices points of a rectangle.
6. (We know that opposite side of a rectangle and diagonal are equal)
7. Prove that (2, -2), (-2, 1) and (5, 2) are the vertices of a right angled triangle. Find the area of the triangle and the length of the hypotenuse.
8. Prove that the points (2a, 4a), (2a, 6a) and (2a + √3 a , 5a) are the vertices of an equilateral triangle.8. Prove that the point (-2, 5), (0, 1) and (2, -3) are collinear.
9. Which point on y-axis is equidistant from (2, 3) and (-4, 1) ?
10. Find the value of k, if the point P (0, 2) is equidistant from (3, k) and (k, 5).
11. Prove that the points (0, 0), (5, 5) and (-5, 5) are the vertices of a right isosceles triangle.
12. If the point P (k – 1, 2) is equidistant from the points A (3, k) and B (k, 5), find the values of k.
13. If A (3, y) is equidistant from points P (8, -3) and Q (7, 6), find the value of y and find the distance AQ.
14. If the point P (2, 2) is equidistant from the points A (-2, k) and B (-2k, -3), find k. Also, find the length of AP.
15. Find the points of trisection of the line segment joining the points (5, -6) and (-7, 5).
16. Find the ratio in which the point P (x, 2) divides the line segment joining the points A (12, 5) and B (4, -3). Also, find the value of x.
17. Find the coordinates of a point A, where AB is a diameter of the circle whose centre is (2, -3) and B is (1, 4).
18. In what ratio does the point (-4, 6) divide the line segment joining the points A (-6, 10) and B (3, -8)?
19. Find the centroid of the triangle whose vertices are (1, 4), (-1, -1), (3, -2).
20. Two vertices of a triangle are (1, 2), (3, 5) and its centroid is at the origin. Find the Co-ordinates of the third vertex.

SOLUTIONS

1. SOLUTION: $2\sqrt{2}b$
2. SOLUTION: a=4,-2
3. Prove yourself
4. 4.SOLUTION: x=0,y=$\pm\sqrt{7}$
5. Prove yourself
6. 6.SOLUTION: Area=12.5 square unit, length of the hypotenuse=5$\sqrt{2}$ unit
7. Prove yourself
8. Prove yourself
9. SOLUTION:(0,-1)
10. 10. SOLUTION: K=1
11. Prove yourself
12. 12.SOLUTION: K=1,5
13. 13 SOLUTION: $\sqrt{41}$
14. 14. SOLUTION: K=-1,-3 and length of AP=5
15. SOLUTION: (1,$\frac{-7}{3}$),(-3,$\frac{4}{3}$)
16. SOLUTION:x=9
17. SOLUTION: (3,-10)
18. SOLUTION: 2:7
19. SOLUTION: (1,$\frac{1}{3}$)
20. SOLUTION: (-4,-7)

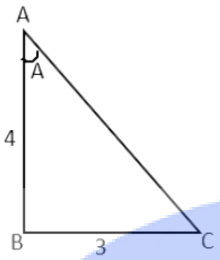
8. INTRODUCTION TO TRIGONOMETRY

1 In ΔABC right angled at B, if tan A $= \frac{1}{\sqrt{3}}$, find the values of:

i) sin A cos C + cos A sin C

ii) cos A cos C – sin A sin C

2



In the given figure find *tan A – cot C*

3 If ∠A and ∠B are acute angles such that *cos A = cos B*, then show that ∠A = ∠B

4 Prove the following:

$2(\cos^4 60^\circ + \sin^4 30^\circ) - (\tan^2 60^\circ + \cot^2 45^\circ) + 3\sec^2 30^\circ = \frac{1}{4}$

5 For a right angled triangle, prove:

$\sin^2\theta + \cos^2\theta = 1$

6 For a right angled triangle, prove:

$1 + \tan^2\theta = \sec^2\theta$

7 For a right angled triangle, prove:

$1 + \cot^2\theta = \mathrm{cosec}^2\theta$

8 Show that $(1 + \tan A \tan B)^2 + (\tan A - \tan B)^2 = \sec^2 A \sec^2 B$

9 Prove that: $\frac{\sin\theta}{1-\cos\theta} + \frac{\tan\theta}{1+\cos\theta} = \sec\theta.\mathrm{cosec}\,\theta + \cot\theta$

10 Find the value of θ in the following:

$\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = 4$

11 Find the value of θ in the following:

$\frac{\cos\theta}{1+\mathrm{cosec}\,\theta} + \frac{\cos\theta}{\mathrm{cosec}\,\theta - 1} = 2$

12 Prove that:

$\frac{\cot A + \tan B}{\cot B + \tan A} = \cot A.\tan B$

13 Prove the following:

$\frac{1}{cosec\,\theta - \cot\theta}$ - $\frac{1}{\sin\,\theta}$ = $\frac{1}{\sin\,\theta}$ - $\frac{1}{cosec\,\theta + \cot\theta}$

14 Prove the following: $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$ = 2 sec θ

15 Show that: $\frac{\sin A}{\sec A + \tan A - 1} + \frac{\cos A}{cosec\,A + \cot A - 1}$ = 1

16 Write the other trigonometric ratios in terms of *sec A*

17 If 3 cot A = 4, then check whether $\frac{1-tan^2 A}{1+tan^2 A}$ = $\cos^2$ A – $\sin^2$ A or not.

18 If *cos ϑ + sin ϑ = √2 cos ϑ*, show that *cos ϑ – sin ϑ = √2 sin ϑ*

19 If *sin ϑ + cos ϑ = √3*, then prove that *tan ϑ + cot ϑ = 1*

20 Given *cos ϑ* = $\frac{21}{29}$, determine the value of $\frac{\sec\theta}{\tan\theta - \sin\theta}$

HINTS

1 For ∠A, we have perpendicular and base. We need to calculate the hypotenuse using the Pythagoras property.
Then we can calculate the t-ratios for ∠*A* and ∠*C*
Put the values to find out the answers

2 Find AC by Pythagoras property
Then find the values of tan A and cot C

3 Consider 2 triangles PQA and RSB
cos A = cos B (given)
Prove similarity for both the triangles, to get the result

4 Put the values of all the trigonometric ratios

5 Consider a right angled triangle. Find out the values of sin θ and cos θ in terms of the sides. Put the values in the LHS.

6 Consider a right angled triangle. Find out the values of tan θ and sec θ in terms of the sides. Put the values in the LHS.

7 Consider a right angled triangle. Find out the values of cot θ and cosec θ in terms of the sides. Put the values in the LHS.

8 Use the identity $(a + b)^2$ and $(a – b)^2$ on the LHS

9 Take LCM on the LHS and solve

10 Solve the LHS until you get the value of cos θ (=*1/2*)

11 Solve the LHS until you get the value of tan θ (=*1*)

12 Convert the LHS into sin and cos, then proceed

13 From the question, put both the fractions with sin θ on one side and the remaining 2 fractions on the other side. Then proceed with LHS = RHS

14 On the LHS, solve both the square roots by multiplying the opposite signs of the denominators in numerator and denominator.

15 Take LCM on the LHS. Then convert *cosec*, *cot* and *tan* into *sin* and *cos*

16 Consider a right angled triangle ABC with $\angle B = 90^{o}$
Find out the value of sec A by *Hyp/Base*
Consider the sides in the form of *sec A*

17 From the given value of cot A, find the other trigonometric ratios and then check for the possibility.

18 Square both the sides of the given equation

19 Convert *tan ϑ* and *cot ϑ* into *sin ϑ* and *cos ϑ*
Then take LCM and proceed

20 Find the values of *sec ϑ, tan ϑ* and *sin ϑ* after applying the Pythagoras property and find the desired value.

9. SOME APPLICATIONS OF TRIGONOMETRY

1. The angle of elevation of an aeroplane from a point on the ground is 60^0. After a flight of 30 seconds the angle of elevation becomes 30^0. if the aeroplane is flying at the height of 3000√3m, find the speed of the aeroplane.

2. The angle of depression of the top and the bottom of an 8 m tall building from the top of a multi-storeyed building are 30^0 and 45^0, respectively. Find the height of the multi-storeyed building and distance between two buildings.

3. The angle of elevation of a cloud from a point 120m above a lake is 30^0 and the angle of depression of it's reflection in the lake is 60^0. find the height of the cloud from the surface of the water.

4. The angle of the elevation of the top of the tower from two points P and Q at distances of a and b respectively from the base and in the same base line with it are complementary. Prove that the height of the tower is √ab.

5. A boy observes that the angle of elevation of the bird flying at a distance of 100m is 30^0. at the same distance from the boy, a girl finds the angle of elevation of the same bird from a building 20m high is 45^0. find the distance of the bird from the girl.

6. The shadow of a flag staff is 3 times as long as the shadow of the flagstaff when the sun rays meet the ground at an angle of 60^0. Find the angle between the sunrays and the ground at the time of longer shadow.

7. The length of the shadow of the tower standing on level plane is found to be 2x meters longer when the sun's altitude is 30^0 than when it was 45^0. prove that the height of the tower is x(√3+1).

8. A fire in the building B ia reported on telephone to two fire stantions P and Q, 20km apart from each other on a straight road. P observes that the fire is at an angle of 60^0 to the road and q observes that it is an angle of 45^0 to the road. Which station should send its team and how much will this team have to travel.

9. The angle of depression of the top and bottom of a 7m tall building from the top of a tower are 45^0 and 60^0 respectively. Prove that the height of the tower is 7((1+√3)/2)m.

10. A statue, 1.6m tall, stands on the top of the pedestal. From a point on the ground. The angle of elevation of the top of the statue is 60^0 and from the same point the angle of elevation of the top of the pedestal is 45^0 . find the height of the pedestal. (4/5(√3+1))

11. From the top of the tower 50m high the angle of depression of the top and bottom of a pole are observed to be 45^0 and 60^0 respectively. Find the height of the pole if the pole and the tower stands on same plane. [(1/√3)=0.577] (21.15m)

12. The angle of elevation of the top of the chimney from the foot of the tower is 60^0 and the angle of depression of the foot of the chimney from the top of the tower is 30^0. if the height of the tower is 40m. Find the height of a smoke emitting chimney. According to pollution control norms , the minimum height of the smoke emitting chimney should be 100m . what value is discussed in this question?(40m)

13. A moving boat is observed from the top of a 150m high cliff moving away from the cliff. The angle of depression of the boat changes from 60^0 to 45^0 in 2 minutes. Find the speed of the boat in m/hr. (1902m/hr)

14. There are two temples, one on each bank of the river, just opposite to each other. One temple is 50m high. The angle of depression of the top and the foot of the other temple are 30^0 and 60^0 respectively. Find the width of the river and the height of the other temple.

15. A pole of height 5m is fixed on the top of the tower. The angle of elevation of the top of the pole as observed from a point A on the ground is 60^0 and the angle of depression of the point A from the top of the tower is 45^0. Find the height of the tower. ($\sqrt{3}=1.732$)

16. The angle of elevation of a cloud from a point 60m above a lake is 30^0 and the angle of depression of the reflection of the cloud in the lake is 60^0. Find the height of the cloud from the surface of the lake.

17. A 1.2m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2m from the ground. The angle of elevation of the balloon from the eyes of the girl at that instant is 60^0. After sometimes angle of elevation reduce to 30^0 . find the distance travelled by the balloon during the interval. 98

18. At the foot of the mountain, the elevation of its summit is 45^0 . After ascending 1000m towards the mountain up a slope tower is 30^0 . find the height of the tower and the width of the canal.

19. A man sitting at a height of 20m on a tall tree on a small island in the middle of the river observes two poles directly opposite each other on the two banks of the river and in a line with the foot of the tree. If the angle of the depression of the feet of the poles from a point at which the man is sitting on the tree on either side of the river are 60^0 and 30^0 respectively. Find the width of the river.

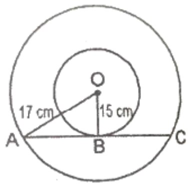
20. the lower window of the house is at a height of 2m above the ground and its upper window is 4m vertically above the lower window. At certain instant the angles of elevation of a balloon from these windows are observed to be 60^0 and 30^0 respectively. Find the height of the balloon above the ground.

HINTS

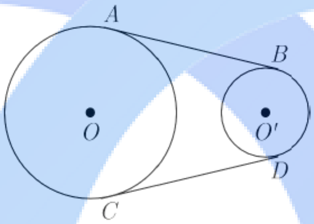
Do it yourself.

10. CIRCLES

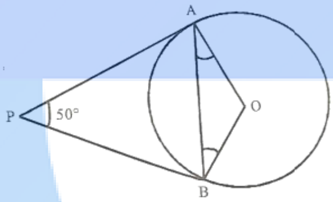
1. Q1. If radii of two concentric circles are 15cm and 17cm, then find the length of the chord of one circle which is tangent of the other.



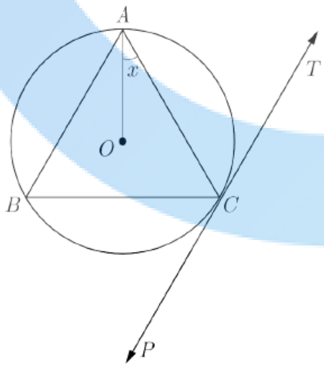
2. If angle between two tangents drawn from a point P to a circle of radius a and centre O is 60^0, then find the length of AP.
3. In the figures AB and CD are common tangents to circles of unequal radii. Prove that AB = CD



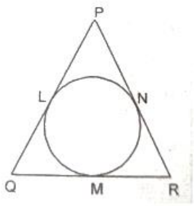
4. In the given figure PA and PB are two tangents to the circle with centre (o) such that the angle APB is 50^0. Find the measure of angle OAB.



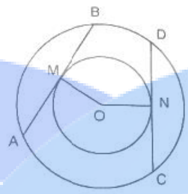
5. In the adjoining figure, PT is a tangent at point C of the circle with centre O. If $\angle ACP = 118^\circ$, then find the measure of $\angle x$.



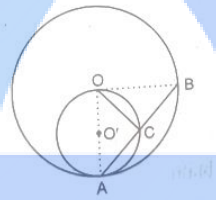
6. In the figure if PQ = PR, show that QM = MR.



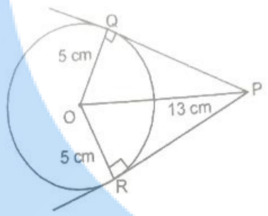
7. In two concentric circles, prove that all chords of the outer circle which touch the inner circle are of equal length.



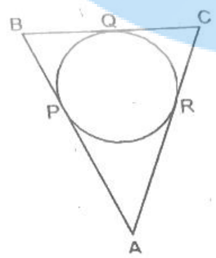
8. In the given figure, C(O,r) and C(O', r/2) touch internally at point A and AB is a chord of the circle C(O,r)intersecting C(O',r/2)at C.Prove that AC = CB.



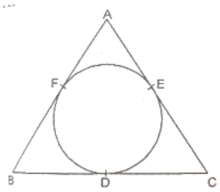
9. From a point P, which is at a distance of 13 cm from the centre of a circle of radius 5 cm, the pair of tangents PQ and PR drawn to the circle. Find the area of the quadrilateral PQOR. (in cm^2.)



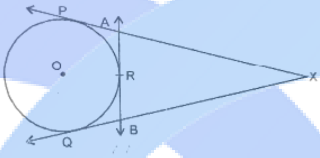
10. In the given figure, if AP= 5cm, BQ=2 cm, CR = 3 cm then find the perimeter of triangle ABC.



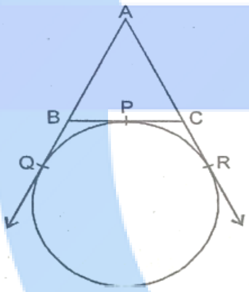
11. In the given figure, the incircle of triangle ABC touches the sides, BC, CA and AB at D, E, F respectively. Show that AF + BD + CE = half perimeter of triangle ABC.



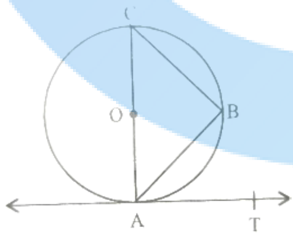
12. In the given figure XP, XQ are tangents from X to the circle with centre O.R is a point on the circle. Prove that XA + XR = XB + BR



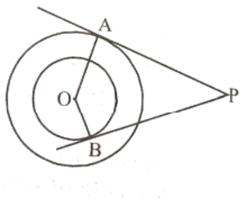
13. In the given figure a circle touches the side BC of triangle ABC at P and touches AB, AC produced at Q and R respectively. If AQ = 5 cm, find the perimeter of triangle ABC.



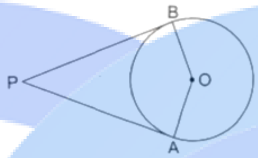
14. If AB is a chord of a circle with centre O, AOC is a diameter and AT is the tangent at A as shown in the figure, prove that angle BAT = angle ACB



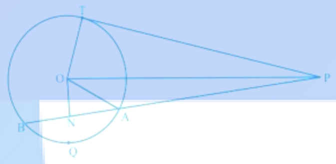
15. Tangents PA, PB are drawn from an external point P to two concentric circles with centre O and radius 8 cm and 5 cm respectively. If AP = 15 cm, find the length of BP.



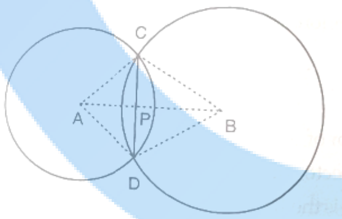
16. If d_1, d_2 ($d_1 > d_2$) be the diameters of two concentric circles, C be the length of the chord of the bigger circle which is also a tangent of the smaller circle prove that $d_2^2 = c^2 + d_1^2$.
17. In the given figure PA, PB are two tangents to a circle with centre O. Show that the points A, O, B, P are concyclic.



18. In the given figure from an external point P, a tangent PT, and a line segment PAB is drawn to a circle with centre as O. ON is perpendicular to the chord AB. Prove that
    $PA \cdot PB = PN^2 - AN^2$
    $PN^2 - AN^2 = OP^2 - OT^2$



19. Two circles with centres A and B with radii 3 cm and 4 cm respectively intersect at points C and D such that AC and BC are tangents to the two circles. Find the length of the common chord CD.

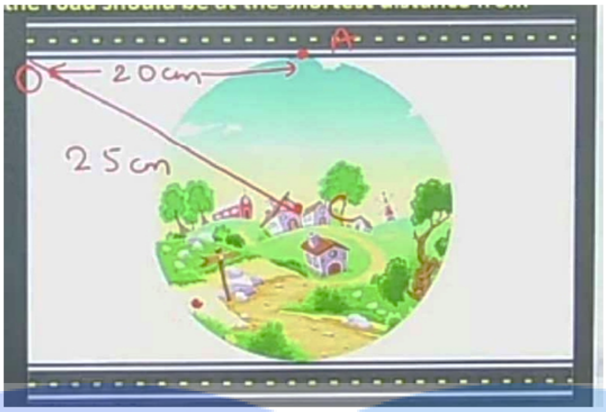


20. CASE STUDY BASED QUESTION.

    PALAMPUR VILLAGE

    People of the village want to construct road nearest to a circular village Rampur. The road cannot pass through the village. But the people want the road to be at shortest distance from the centre of the village. Suppose the road starts from point O, which is outside the village and touch the boundary of the circular village at A, such that OA is 20 cm. Also, the straight distance of the point O from the centre of the village is 25 cm.

Answer the following questions



(i) Find the shortest distance of the road from the centre of the village.
(ii) Which theorem you need to apply to find the shortest distance?
(iii) What will be the area of the village?
(iv) What will be the distance a person has to cover if he has to travel between any two points on the boundary and passing through the centre?
(v) What will be the measure of the angle OAC?

ANSWERS/HINTS

1. 128 cm
2. AP = a$\sqrt{3}$ units
4. 250
5. x = 28°
9. 60 cm^2
10. 20 cm
13. 10 cm
15. 2$\sqrt{66}$ cm
19. 4.8 cm
20.
(i) 15 cm
(ii) Pythagorus theorem
(iii) 706.5 cm^2
(iv) 30 cm
(v) 90^0

11. CONSTRUCTION

1 Draw a pair of tangents to a circle of radius 5 cm which are inclined to each other at an angle of 60°.

2 Let ABC be a right triangle in which AB = 6 cm, BC = 8 cm and . B = 90°. BD is the perpendicular from B on AC. The circle through B, C, D is drawn. Construct the tangents from A to this circle.

3 Construct a tangent to a circle of radius 4cm from a point on the concentric circle of radius 6cm.

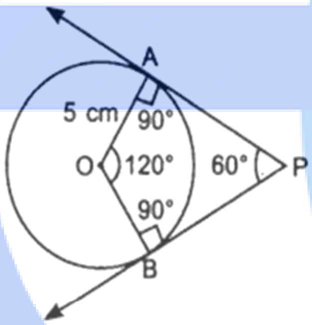
ANSWER

1

1. Draw a circle of radius 5 cm.
2. As tangents are inclined to each other at an angle of 60°.

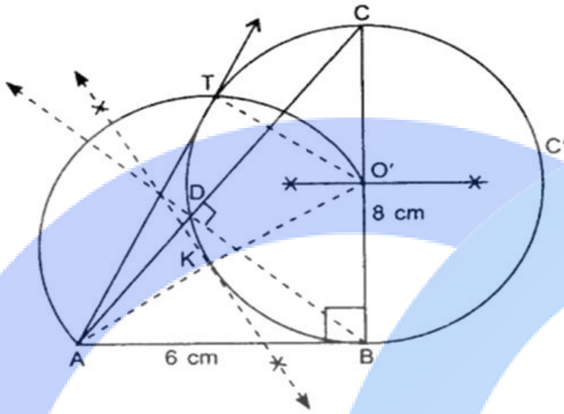
∴ Angle between the radii of circle is 120°. (Use quadrilateral property)

3. Draw radii OA and OB inclined to each other at an angle 120°.
4. At points A and B, draw 90° angles. The arms of these angles intersect at point P.
5. PA and PB are the required tangents.

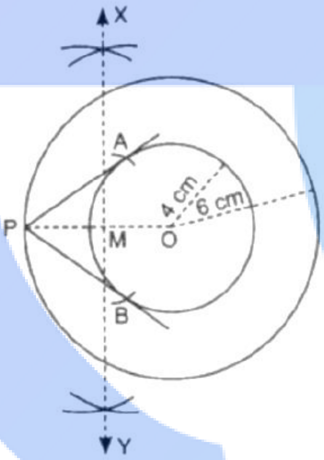


2

1. Draw a right triangle ABC with AB = 6 cm, BC = 8 cm and ∠B = 90°.
2. From B, draw BD perpendicular to AC.
3. Draw perpendicular bisector of BC which intersect BC at point O'.
4. Take O' as centre and O'B as radius, draw a circle C' passes through points B, C and D.
5. Join O'A and draw perpendicular bisector of O'A which intersect O'A at point K.
6. Take K as centre, draw an arc of radius KO' intersect the previous circle C' at T.
7. Join AT, AT is required tangent.



3.

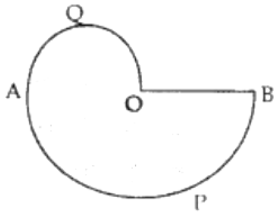


Steps of construction:

1. Draw two concentric circles with centre O of radii 4 cm and 6 cm.
2. Take a point P on the bigger circle of radius 6 cm.
3. Join OP with dotted line.
4. Draw perpendicular bisector of OP which intersects OP at M.
5. With M as centre and MP radius, mark two arcs on smaller circle of radius 4 cm at point A & B.
6. Join PA and PB.
   PA and PB are the required pair of tangents.

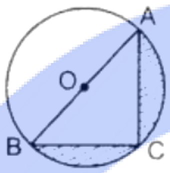
12. AREAS RELATED TO CIRCLES

1. In Figure, APB and AQO are semi-circles, and AO = OB. If the peri-meter of al the figure is 40 cm, find the area of the shaded region. [Use $\pi = 22/7$]



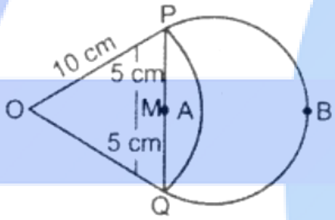
ANS-96.25 cm^2

2. In Figure, O is the centre of a circle such that diameter AB = 13 cm and AC = 12 cm. BC is joined. Find the area of the shaded region. (Take $\pi = 3.14$)

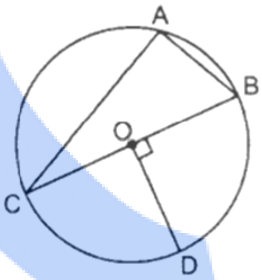


ans-36.33cm^2

3. In Figure, are shown two arcs PAQ and 0 PBQuestion Arc PAQ is a part of circle with centre O and radius OP while arc PBQ is a semi-circle drawn on PQ as diameter with centre M. If OP = PQ = 10 cm, show that area of shaded region is $25(\sqrt{3}-\pi/6)cm^2$.



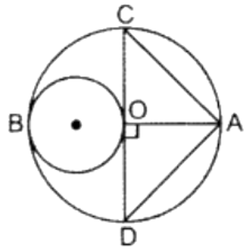
4. In Figure, O is the centre of the circle with AC = 24 cm, AB = 7 cm and ∠BOD = 90°. Find the area of the shaded region. (Use $\pi = 3.14$)



Ans-284cm^2

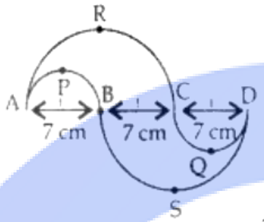
5. In Fig., AB and CD are two diameters of a circle with centre 0, which are perpendicular to each other. OB is the diameter of the smaller circle. If OA = 7 cm, find the area of the shaded region. (Use

π = 22/7)



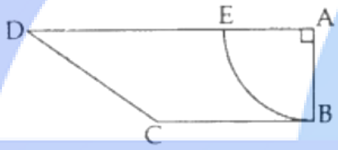
ans-66.5cm^2

6. In Figure, APB and CCD are semi-circles of diameter 7 cm each, while ARC and a BSD are semi-circles of diameter 14 cm each. Find the perimeter of the shaded region. [Use π = 22/7]



ans-66cm^2

7. In Figure, ABCD is a trapezium of area 24.5 sq. cm. In it, AD || BC, ∠DAB = 90°, AD = 10 cm and BC = 4 cm. If ABE is a quadrant of a circle, find the area of the shaded region. [Take π = 22/7]

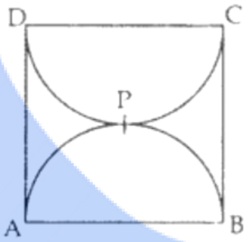


ans-14.875cm^2

8. Find the area of the minor segment of a circle of radius 14 cm, when its central angle is 60°. Also find the area of the corresponding major segment. [Use π = 22/7]

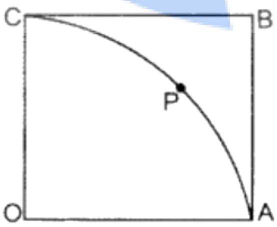
Ans-616cm^2

9. Find the perimeter of the D shaded region in Figure, if ABCD is a square of side 14 cm and APB and CPD are semicircles. [Use π = 22/7]



ans-72cm

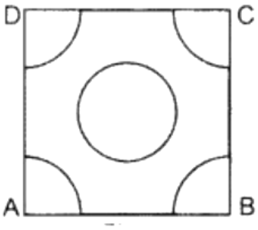
10. In Figure, OABC is a square of side 7 cm. If OAPC is a quadrant of a circle with centre O, then find the area of the sha- o ded region. [Use π = 22/7]



ans-10.5cm^2

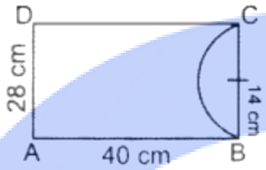
11. In Figure ABCD is a square of side 4 cm. A quadrant of a circle of radius 1 cm is drawn at each vertex of the square and a circle of diameter 2 cm is also drawn. Q12.Find the area of the shaded region.

(Use π = 3.14)



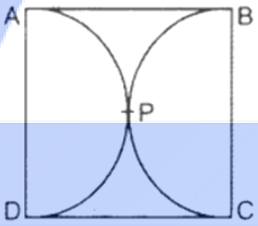
ans-68/7 cm

12. From a rectangular sheet of paper ABCD with AB = 40 cm and AD = 28 cm, a semi-circular portion with BC as diameter is cut off. Find the area of the remaining paper. (Use π = 22/7)



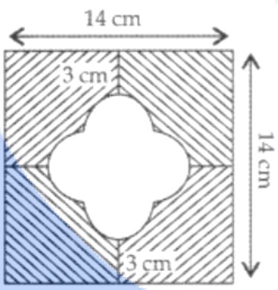
Ans-812cm^2

13. In Figure, find the area of the shaded region, if ABCD is a square of side 14 cm and APD and BPC are semi-circles.



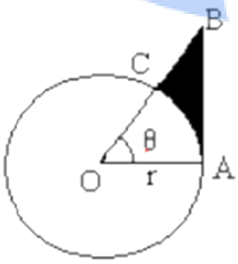
ans-42cm^2

14. In Figure, find the area of the shaded region. [Use π = 3.14]



ans-25.12cm^2

15. In figure, shows a sector of a circle, centre O, containing an angle θ



Prove that

Perimeter of shaded region is

$r\,[\tan\theta + \sec\theta + \frac{\pi\theta}{180} - 1]$

Area of shaded region is

$\frac{r^2}{2}[\tan\theta - \frac{\pi o}{180} - 1]$

16. Find the number of revolutions made by a circular wheel of area 1.54 m^2 in rolling a distance of 176 m.

17. Find the area of the sector of a circle with radius 4cm and of angle 30°. Also, find the area of the corresponding major sector.

HINT- Area of sector = $[\theta/360]\times\pi r^2$

Area of major sector = $((360 - \theta)/360)\times\pi r^2$

Ans- 46.05 cm^2, 4.19 cm^2

18. Calculate the perimeter of an equilateral triangle if it inscribes a circle whose area is 154 cm^2

HINT- r = Area of triangle/semi-perimeter ans- 72.7 cm.

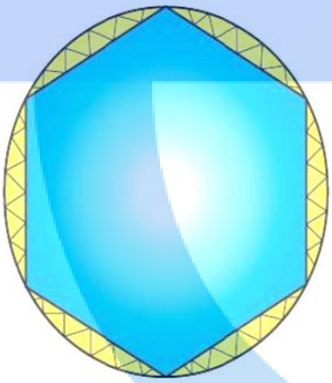
19. The wheels of a car are of diameter 80 cm each. How many complete revolutions does each wheel make in 10 minutes when the car is travelling at a speed of 66 km per hour?

HINT- in one revolution, the distance covered = circumference of the wheel

the no. of revolutions of the wheels = (Distance covered by the car/Circumference of the wheels

ans- 4375.

20. A round table cover has six equal designs as shown in the figure. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of Rs. 0.35 per cm^2. (Use 3 = 1.7)



HINT- AOB = 360°/6 = 60°

Area of equilateral ΔAOB = $\sqrt{3}/4 \times a^2$

Ans- Rs. 162.66

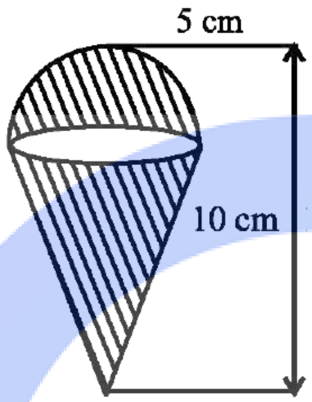
21. Area of the largest triangle that can be inscribed in a semi-circle of radius r units is.

HINT- The largest triangle that can be inscribed in a semi-circle of radius r units is the triangle having its base as the diameter of the semi-circle

Ans. r^2 sq. units

13.SURFACE AREAS AND VOLUMES

1. A solid sphere of radius r is melted and recast into the shape of a solid cone of height r. Find radius of the base of the cone.
2. A cylinder and a cone are of same base radius and of same height. Find the ratio of the volumes of cylinder to that of the cone.
3. An ice cream cone full of ice cream having radius 5 cm and height 10 cm as shown. Calculate the volume of ice cream (to the nearest integer, in cm3), provided that its (1/6)th part is left unfilled with ice cream. Insert answer in nearest integer.



4. Marbles of diameter 1.4 cm are dropped into a cylindrical beaker of diameter 7 cm containing some water. Find the number of marbles that should be dropped into the beaker so that the water level rises by 5.6 cm.
5. How many spherical lead shots each of diameter 4.2 cm can be obtained from a solid rectangular lead piece with dimensions 66 cm, 42 cm and 21 cm.
6. How many spherical lead shots of diameter 4 cm can be made out of a solid cube of lead whose edge measures 44 cm.
7. A wall 24m long, 0.4m thick and 6m high is constructed with the bricks each of dimensions 25cm*16cm*10cm. If the mortar occupies (1/10)th of the volume of the wall, then find the number of bricks used in constructing the wall.
8. Find the number of metallic circular disc with 1.5 cm base diameter and of height 0.2 cm to be melted to form a right circular cylinder of height 10 cm and diameter 4.5 cm.
9. A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 4 cm and the diameter of the base is 8 cm. Determine the volume of the toy. If a cube circumscribes the toy, then find the difference of the volumes of cube and the toy. Also, find the total surface area of the toy.
10. A solid metallic hemisphere of radius 8 cm is melted and recasted into a right circular cone of base radius 6 cm. Determine the height of the cone.
11. Find the height of largest right circular cone that can be cut out of a cube whose volume is 729 cm^3.
12. Two identical cubes each of volume 64 cm^3 are joined together end to end. What is the surface area of the resulting cuboid?
13. Twelve solid spheres of the same sizes are made by melting a solid metallic cylinder of base diameter 2 cm and height 16cm. Find the radius of each sphere.
14. Three cubes of a metal whose edge are in the ratio 3:4:5 are melted and converted into a single cube whose diagonal is 12cm. Find the edge of three cubes.
15. What cross-section is made by a cone when it is cut parallel to its base?
16. Find total surface area of a solid hemi-sphere of radius 7cm.

17. Volume of two spheres is in the ratio 64:125. Find the ratio of their surface areas.
18. A rectangular water tank of base 11m ×6m contains water upto a height of 5m. If the water in the tank is transferred to a cylindrical tank of radius 3.5m, find the height of the water level in the tank in cm.
19. How many cubic cm of iron is required to construct an open box, whose external dimensions are 36 cm, 25 cm and 16.5 cm, provided the thickness of the iron is 1.5 cm. If one cubic cm of iron weight 7.5 g. Find weight of the box.
20. The barrel of a fountain-pen, cylindrical in shape, is 7 cm long and 5 mm in diameter. A full barrel of ink in the pen will be used up on writing 330 words on an average. How many words would use up a bottle of ink containing one fifth of a litre?
21. Water flows at the rate of 10 m per minute through a cylindrical pipe having its diameter as 5 mm. How much time will it take to fill a conical vessel whose diameter of base is 40 cm and depth 24 cm.

SOLUTIONS

1. Sol 1:Volume of Cone= Volume of Sphere
   Let h=R
   $1/3\ \pi r^2 h = 4/3\pi r^3$
   $1/3\ \pi r^2 R=4/3\pi r^3$
   R=2r
2. Sol 2.
   [hint: 3 : 1 : 2]
3. Sol 3:
   Volume of ice cream = Volume of hemisphere + Volume of cone
   $=2/3\pi r^3+1/3\pi r^2\ h$
   Radius of hemisphere = Radius of cone
   Since height of hemisphere is 5 cm, then height of cone will be 10−5=5cm
   ∴ Volume of ice cream
   $=2/3\pi(5)^3+1/3\ \pi(5)^2 \times 5$
   =125π=392.85
   1/6th of ice cream =692.85/6 =65.475
   Volume of required portion of ice cream =392.85−65.47=327.375cm3
4. Sol 4:
   Diameter of marble =1.4cm
   Therefore,
   radius of marble =21.4=0.7cm
   Volume of 1 marble $=4/3\pi r^3=4/3\times\pi\times(0.7)^3$.....(1)
   Now,
   Diameter of beaker =7cm
   Therefore,
   radius of beaker =27=3.5cm
   Height of water =5.6cm
   Volume of water =πr2h$=\pi\times(3.5)^2\times5.6$.....(2)
   Now,
   No. of marbles dropped =Volume of 1 marbleVolume of water
   ⇒ No. of marbles dropped $=(\pi\times(3.5)^2\times5.6\ )/(4/3\times\pi\times(0.7))$
   ⇒ No. of marbles dropped =150

5. Sol 5:
   Volume of Cuboid = 58212cm^3
   Volume of Sphere = 38.77cm^3
   N= vol of cuboid / vol of Sphere = 1500
6. Sol 6:
   Vol of Sphere = 33.52cm^3
   Vol of cube= 85184 cm^3
   N= vol of cube / vol of Sphere = 2541
7. Sol 7:
   Vol of wall =57.6 m^3
   Vol of one Brick =0.004 m^3

   Since 1/10th of the volume of the wall is occupied by mortar, so the volume of bricks in the wall, =(1−1/10) part of the wall,

   1/10th vol of wall =51.84 m^3
   N= Vol of Bricks in wall/ Vol of one brick=1296
8. Sol 8:
   Volume of cylinder =n× Volume of circular disc
   n= 450
9. Sol 9.
   Volume of the cube = 512 cm^3
   Volume of the toy = volume of the hemisphere + volume of the cone
   = 1408/7cm^3
   Difference in the volumes of the cube and the toy= 310.86 cm^3
   Total surface area of the toy = curved surface area of cone + curved surface area of hemisphere
   = 171.68 cm^2
10. Sol 10: Volume of hemisphere = Volume of cone
    h= 28.44cm
11. Sol 11:
    Side of cube = 9cm
12. Sol 12:
    Side of cube = 4 cm
    Length, breadth and height of new cuboid is 8 cm, 4 cm and 4 cm respectively. Surface area of cuboid = 2[8 × 4 + 4 × 4 + 4 × 8] = 160 cm^2
13. Sol 13:
    Volume of 12 solid sphere = Volume of solid cylinder

    $12 \times \frac{4}{3} \pi r^3 = \pi(1)^2 \times 16$
    $r^3 = 1$
    r = 1 cm
14. sol 14:
    Let the edges of three cubes be 3x cm, 4x cm and 5x cm. Volume of single cube = Sum of volume of three cubes (Side)^3 = $(3x)^3 + (4x)^3 + (5x)^3$

Side = 6x cm

Diagonal of single cube = 12√3

x = 2

Hence edges of three cubes are 6 cm, 8 cm and 10 cm

15. Sol 15:
16. Sol 16: 3�r^2
17. Sol 17: 3 units
18. Sol 18:

    Volume of cuboidal tank =(11)(6)(5)= 330 cu. m

    Volume of cylindrical tank =38.465h

    Volume of cuboidal tank = Volume of cylindrical tank

    ∴h=38.465330=8.579cm
19. Sol 19:

    External vol = 36×25×16.5=14850 cu. cm

    Thickness of iron =1.5 cm

    Internal l = 36–2(1.5) =33 cm

    b=25–2(1.5)=22 cm

    h=16.5–2(1.5)=13.5 cm

    Internal =l×b×h

    Volume =33×22×13.5

    Volume of iron used =(external -internal) volume

    =(14850–9801) cu cm

    5049 cu cm

    Weight of 1 cu cm of iron =7.5 gm

    Then, weight of the box =5049×7.5

    37867.5 gms

    37.86 kg.
20. Sol 20:

    Volume of a barrel = 1.375 cu cm

    Volume of ink in the bottle = 1/5 litre = 1000/5= 200 cu cm

    Therefore, total number of barrels that can be filled from the given volume of ink = 200/1.375

    So, required number of words = 200/1.375×330 = 48000
21. Sol 21:

    Amount of water required to fill the conical vessel = volume of conical vessel

    3200 π cu.cm .

    Amount of water that flows out of cylindrical pipe in 1 minute

    62.5 π cu.cm

    Time required to fill the vessel = 3200/62.5 = 51.2 min

14. STATISTICS

Question 1

For the following frequency distribution:

Class	10-20	20-30	30-40	40-50	50-60
Frequency	6	5	8	25	20

a. What is the lower limit of frequency 20?
b. Write the class mark of above frequency distribution.
c. What is the upper limit of frequency 5?
d. Find $\sum$ Fi.

Question 2

Answer the following from given data

class interval	15-30	30-45	45-60	60-75	75-90
Frequency	6	18	19	4	11

a. identify the modal class
b. upper limit of modal class
c. frequency of modal class
d. frequency of class succeeding the modal class
e. frequency of class preceding the modal class

Question 3

Find the value of x if sum of frequencies is 40 and complete cumulative frequencies.

Class interval	Frequency	Cumulative frequency
10-20	5	
20-30	8	
30-40	15	
40-50	X	
50-60	10	
	N=40	

Question 4

Find the mean of following distribution by assumed mean method.

Marks	No. Of students
Below 10	4
Below 20	10
Below 30	18
Below 40	28
Below 50	40
Below 60	70

Question 5

The mean of the following data is 50. Find the missing frequencies p_1 and p_2

C.I.	frequency
0-20	17
20-40	p_1
40-60	32

60-80	p_2
80-100	19
Total	120

Question 6

Choose correct answer from the following frequency distribution

Class interval	10-20	20-30	30-40	40-50	50-60	60-70
No. of students	5	10	15	20	25	30

1. Upper limit for frequency 25.
   a.50 b.60 c.55 d.70
2. Modal class is:
   a. 40-50 b. 60-70 c. 50-60 d. 10-20
3. Frequency of modal class is:
   a. 25 b. 30 c.40 d.5
4. Size of the class interval is:
   a.8 b.15 c.10 d.20
5. Median class is:
   a. 50-60 b. 60-70 c.40-50 d.30-40

Question 7

Find the mean of the following data

Classes	frequency
0.5-5.5	13
5.5-10.5	16
10.5-15.5	22
15.5-20.5	18
20.5-25.5	11

Question 8

The median of the distribution given below is 14.4. find the values of x and y if the total frequency is 20.

Class interval	0-6	6-12	12-18	18-24	24-30
frequency	4	X	5	y	1

Question 9

For the following frequency distribution find the mode.

Class	3-6	6-9	9-12	12-15	15-18	18-21	21-24
Frequency	2	5	10	23	21	12	3

Question 10

Fill the missing numbers in the given frequency distribution.

Class interval	Frequency	Class mark	Cumulative frequency
5-10	4		4
10-15	5	12.5	
15-20	2		11
20-25	12	22.5	

25-30	3		26

Question 11

Find the frequencies of the distribution when cumulative frequencies are given

Class	Cumulative frequency
10-20	5
20-30	12
30-40	15
40-50	20
50-60	30

Question 12

Find the value of (x+y) for following distribution when sum of all frequencies is 80.

Class	frequency	Cumulative frequency
5-10	10	
10-15	X	
15-20	20	
20-25	Y	
25-30	15	
total	80	

Question 13

Find the missing frequencies in the following distribution, if the sum of frequencies is 120 and the mean is 50.

Class	0-20	20-40	40-60	60-80	80-100
frequency	17	F1	32	F2	19

Question 14

Find the mean of following distribution by assumed mean method

Class interval	frequency
10-20	2
20-30	5
30-40	3
40-50	10
50-60	6

Question 15

Calculate the median from the following data

Marks	0-10	10-20	20-30	30-40	40-50
No. of students	8	16	36	34	6

Question 16

Complete the table 2 using table 1

Table 1

Classes	frequency
10-20	5
20-30	2
30-40	8
40-50	6
50-60	3
60-70	4

Table 2		
Cumulative frequency	Modal class =	Median class =
	Upper limit of modal class=	Cumulative frequency of the class preceding the median class=
	Lower limit of modal class =	frequency of the median class =
	Frequency of modal class =	Class size=

Question 17

The following table gives the literacy rate (in percentage) of 35 cities. Find the mean literacy rate

Literacy rate(in %)	45-55	55-65	65-75	75-85	85-95
Number of cities	3	10	11	8	3

Question 18

Find the class mark, modal class and median class for following data and also find median of data

Class	4-8	8-12	12-16	16-20
frequency	2	11	4	3

Question 19

Find the median of following data.

Height(in cm)	No. Of students
Less than 120	12
Less than 140	26
Less than 160	34
Less than 180	40
Less than 200	50

Question 20

While computing mean of grouped data, we assume that the frequencies are

a. Evenly distributed over all the classes.
b. Centred at the class marks of the classes.
c. Centred at the upper limit of the classes.
d. Centred at the lower limit of the classes

ANSWER

Ans. 1

Class	10-20	20-30	30-40	40-50	50-60
Frequency	6	5	8	25	20

a. 50
b. 15, 25, 35, 45, 55
c. 30
d. 64

Ans. 2

Class interval	15-30	30-45	45-60	60-75	75-90
Frequency	6	18	19	4	11

a. 45-60
b. 60
c. 19
d. 4
e. 18

Ans. 3

Here given ∑Fi= 40 find the value of x and cumulative frequency is

Class interval	Frequency	Cumulative frequency
10-20	5	5
20-30	8	13
30-40	15	28
40-50	X=2	30
50-60	10	40
	N=40	

Ans. 4 Frequency Distribution is as follow

Marks	Xi	Fi	Di=Xi-a	FiDi
0-10	5	4	-30	-120
10-20	15	6	-20	-120
20-30	25	8	-10	-80
30-40	35=a	10	0	0
40-50	45	12	10	120
50-60	55	30	20	600
		∑Fi=70		∑Fidi=400

By formula of assumed mean method;

Mean= a+ ∑Fidi/∑Fi

35+400/70

35+5.7= 40.7

Ans. 5 Given ∑Fi= 120 and Mean= 50

Class	0-20	20-40	40-60	60-80	80-100	

Frequency(Fi)	17	P1	32	P2	19	∑Fi= 68+p1+p2
Xi	10	30	50	70	90	
FiXi	170	30P1	1600	70P2	1710	∑FiXi=3480+30P1+70P2

Here Sum of frequencies is given as 120 so ∑Fi=120

68+P1+P2= 120

P1 + P2= 52 .... (1)

Using Mean= $\sum F_iX_i / \sum F_i$

(3480+30P1+70P2)/120=50

3480+30P1+70P2= 6000

30P1+70P2= 2520 ....(2)

On Solving eq. 1 and 2 we get P1= 28 and P2 =24

Ans. 6

Class interval	10-20	20-30	30-40	40-50	50-60	60-70
No. of students	5	10	15	20	25	30

1. Option b
2. Option c
3. Option a
4. Option c
5. Option a

Ans. 7

Classes	Xi	Frequency(Fi)	FiXi
0.5-5.5	3	13	39
5.5-10.5	8	16	128
10.5-15.5	13	22	286
15.5-20.5	18	18	324
20.5-25.5	23	11	253
		∑Fi=80	∑FiXi=1030

We know Mean= ∑FiXi/∑Fi

= 1030/80= 12.9

Ans. 8

Given Median= 14.4, N=20 and we have the value of x and y

Class interval	0-6	6-12	12-18	18-24	24-30
Frequency	4	x	5	y	1
Cf	4	4+x	9+x	9+x+y	10+x+y

Since N=20 so 10+x+y= 20

X+y=10 .... (1)

We know median = 14.4 so median class is 12-18

So l=12, h= 6, cf= 4+x, f= 5

Median = l+ h x (N/2 –cf)/f

14.4= 12+ 6 x (10-4-x)/5 ……………………………… (2)

On solving eq. 2 we get, x= 4 and putting value of x in eq. 1 we get y= 6

Ans. 9

Class	3-6	6-9	9-12	12-15	15-18	18-21	21-24
Frequency	2	5	10	23	21	12	3

Here modal class is 12-15

F_0=10 f_2=21 h=3 l=12, f_1= 23

As we know that Mode= l+ h x {(f_1-f_0)/2f_1-f_0-f_2)}

12+ 3x {(23-10)/46-10-21}

12 + 3 x 13/15

12+ 2.6 = 14.6

Ans. 10

Class interval	Frequency	Class mark	Cumulative frequency
5-10	4	7.5	4
10-15	5	12.5	9
15-20	2	17.5	11
20-25	12	22.5	23
25-30	3	27.5	26

Ans. 11

Class	Cumulative frequency	Frequency
10-20	5	5(Initial Frequency in cf)
20-30	12	12-5=7
30-40	15	15-12=3
40-50	20	20-15=5
50-60	30	30-20=10

Ans. 12

Given N= 80, we have to find sum of x and y.

Class	Frequency	Cumulative frequency
5-10	10	10
10-15	X	10+x
15-20	20	30+x
20-25	Y	30+x+y
25-30	15	45+x+y
total	80	

We know that sum of frequencies is equal to last term of cumulative frequency

N= 45+x+y

80=45+x+y

X+y= 80-45=35

Ans. 13

Given ∑Fi= 120 and Mean= 50

Class	0-20	20-40	40-60	60-80	80-100	
Frequency(Fi)	17	F1	32	F2	19	∑Fi= 68+F1+F2
Xi	10	30	50	70	90	
FiXi	170	30F1	1600	70F2	1710	∑FiXi=3480+30F1+70F2

Here Sum of frequencies is given as 120 so ∑Fi=120

68+F1+F2= 120

F1 + F2= 52 ……………………………… (1)

Using Mean= $\sum F_iX_i / \sum F_i$

(3480+30F1+70F2)/120=50

3480+30F1+70F2= 6000

30F1+70F2= 2520 ……………………………………………………………(2)

On Solving eq. 1 and 2 we get F1= 28 and F2 =24

Ans. 14

Class interval	Xi	Fi	Di= Xi-a	FiDi
10-20	15	2	-20	-40
20-30	25	5	-10	-50
30-40	35=a	3	0	0
40-50	45	10	10	100
50-60	55	6	20	120
		∑Fi=26		∑FiDi= 130

Using formula of assume mean method

Mean= a+ ∑Fidi/∑Fi

=35+ 130/26 = 35+5 =40

Ans. 15

Marks	0-10	10-20	20-30	30-40	40-50
No. of students	8	16	36	34	6
Cf	8	24	60	94	100

We have to find the median. Here N=100 and N/2=50

Median class=20-30

l= 20, cf= 24, f=36, h= 10

Median = l+ h x (N/2 –cf)/f

=20+ 10 x (50-24)/36

=20 +10 x 26/36

=20+7.2 = 27.2

Ans. 16

Complete the table no. 2 using table no. 1

Table 1	
Classes	Frequency
10-20	5
20-30	2
30-40	8
40-50	6
50-60	3
60-70	4

Table 2		
Cumulative frequency	Modal class=30-40	Median class= 30-40(Because N/2=14, cf greater than 14 is 15 whose class is 30-40)
5	Upper limit of modal class= 40	Cumulative frequency of the class preceding the median class= 7
7	Lower limit of modal class = 30	frequency of the median class = 8
15	Frequency of modal class = 8	Class size= 10
21		
24		
28		

Ans. 17

Literacy rate	Xi	Fi	Fixi
45-55	50	3	150
55-65	60	10	600
65-75	70	11	770
75-85	80	8	640
85-95	90	3	270
		∑Fi=35	∑FiXi= 2430

We know Mean= ∑FiXi/∑Fi

2430/35= 69.43 %

Ans. 18

Class	4-8	8-12	12-16	16-20
frequency	2	11	4	3

Class mark: 6, 10, 14, 18

Modal Class: 8-12

Solution to find Median:

Class	4-8	8-12	12-16	16-20
frequency	2	11	4	3
Cf	2	13	17	20

Here N=20 so N/2=10

Median Class= 8-12

l=8, cf= 2, h= 4, f= 11

Median= l+ h x (N/2 –cf)/f

8+ 4x (10-2)/11

8+ 2.9= 10.9

Ans. 19

Height	Frequency(Fi)	Cf
100-120	12	12
120-140	14	26
140-160	8	34
160-180	6	40
180-200	10	50
	N=50	

Here N/2=25, so median class is 120-140

So l=120, h= 20, f= 14, cf= 12

We know median= l+ h x (N/2 –cf)/f

= 120+ 20 x (25-12)/14

=120+ 18.57 = 138.57

Ans. 20

Option b. centre at the class marks of the classes.

15. PROBABILITY

Question 1. Two dice are numbered 1,2,3,4,5,6 and 1,2,2,3,3,4 respectively. They are thrown And the sum of numbers on them is noted. Find the probability of getting
i) Sum 7 ii) sum is perfect square

Question 2. A jar contains 54 marbles each of which is blue green or white. The probability of selecting a blue marble at random from the jar is 1/3 and probability of selecting a green marble at random is 4/9. How many white marbles are there in the jar?

Question 3. A die is thrown twice. Find the probability that I) 5 may not come either time
ii) Same number may not come on the die thrown two times.

Question 4. Two customers Shy am and Ekta are visiting a particular shop in the same week (Tuesday to Saturday). Each is equally likely to visit the shop on any day as on another day. What is probability that both will visit the shop on i) same day ii) consecutive day

Question 5. Apurva throws two dice once and computes the product of numbers appearing on the dice. Pahe throws one die and squares the number appearing on the dice. Who has a better chance of getting the number 36?

Question 6. Arjun draws a card from a well shuffled deck of playing cards. Find the probability of getting i) a jack of red suit ii) A diamond card iii) 5 or 9 of club

Question 7. Two dice are thrown at the same time what is the probability that the sum of two numbers appearing on the top will be as follows. Complete the table.

Sum on two dice	2	3	4	5	6	7	8	9	10	11	12
Probability	1/36						5/36				1/36

A student argues that there are 11 possible outcomes therefore each of them has a probability 1/11. Do you agree with this argument?

Question 8. A bag contains x numbers of black marbles and some white marbles and a number of white marbles are 2 more than half of the black marbles. If total numbers of marbles are 14 find the probability of a black and also a white marble. (Hint: x + (x/2) + 2 =1)

Question 9. 17 cards numbered 1, 2, 3,.........16,17 are put in a box and mixed thoroughly. One person draws a card from the box. find the probability that the number in the card is i) an odd number ii) a prime number iii) divisible by 3

Question 10. A box contains cards bearing numbers from 6 to 70. If one card is drawn at random from the box, find the probability that it bears i) an odd number less than 30 ii) a composite number between 50 and 70.

Question 11. In a musical game, the person playing music has been advised to stop playing the music at any time within 2 minutes after she starts playing. What is the probability that the music will stop within the first half- minute after starting?

Question 12. A jar contains 24 marbles, some are green and others are blue. If a marble is drawn at random from the jar, the probability that it is green is 2/3. Find the number of blue balls in the jar.

Question 13 If 65% of the population have black eyes, 25% have brown eyes and the remaining have blue eyes. What is the probability that a person selected at random has i) Blue eyes ii) Brown or black eyes iii) Blue or black eyes.

Question 14. A child's game has 8 triangles of which 3 are blue and rest are red, and 10 squares of which 6 are blue and rest are red. One piece is last at random. Find the probability that it is a i) triangle ii) square iii) triangle of red colour

Question 15.Red queens and black jacks are removed from a pack of 52 playing cards. A card is drawn at random from the remaining cards, after reshuffling them. Find the probability that the drawn card is i) a king ii) of red colour iii) a face card iv) a queen

Question 16. A piggy bank contains hundred 50 p coins, fifty 1-rupee coins, twenty 2-rupee coins and ten 5-rupee coins. If it is equally likely that one of the coins will fall out when the bank is turned upside down, find the probability that the coin which fell i) will be of value more than 1 rupee ii) will be of value less than 5 rupees.

Question 17. Cards marked with numbers 2 to 201 are placed in a box and mixed thoroughly. One card is drawn from this box. Find the probability that the number on the card is: i) a prime number less than 20 ii) a number which is a perfect square.

Question 18. A box contains 5 red marbles, 8 white and 4 green marbles. One marble is taken out of the box at random. What is the probability that the marble taken out will be i) a red marble ii) not green?

Question 19. A bag contains 12 balls, out of which x are white. i) if one ball is drawn at random, find the probability that it is a white ball. ii) if 6 more white balls are put in the bag, the probability of drawing a white ball is double than that in (i) find x.

Question 20. An integer is chosen at random from the first two hundred digits. What is the probability that the integer chosen is divisible by 6 or 8?

ANSWERS

Answer 1. When these dice are thrown then total possible outcomes are

Total possible outcomes = 36

i) Sum of number = 7

favourable outcomes are (3,4) (4,3) , (4,3) ,(5,2) ,(5,2) , (6,1) i.e. = 6

Probability that sum of numbers is 7 = 6/36 i.e. ⅙

ii) Sum is a perfect square i.e. sum is 4 or 9

favourable outcomes are (1,3) ,(1,3) ,(2,2) (2,2),(5,4) (6,3) ,(6,3) = 7

P (sum is a perfect square) = 7/36

Answer 2 let number of blue marbles =B and number of green marbles = g

Probability of getting blue marble = b/54 (total number of marbles = 54)

B/54= 1/3 on solving this we get b= 18

Similarly, probability of getting green marble = g/ 54

According to question (g/54) = 4/9 on solving this we get g = 24

Since total marbles are 54 hence number of white marbles = 54-(24+18)

= 12

Answer 3. Total possible outcome = 36

Favourable outcome that 5 will not come either time will be (1,1) (1,2)(1,3)………..= 25 times

Probability that 5 will not come either time =25/36

ii) Probability of getting same number on both cards (1, 1) (2, 2) (3, 3) (4, 4) (5, 5) (6, 6) i.e., = 6/36

So, probability of not getting same number on both cards = 1-(1/6) = 5/6

Answer 4 Total number of days from Tuesday to Saturday = 5

Hence total number of ways both can reach shop = 25,

Possible combinations to reach shop are (Tuesday, Tuesday) (Wednesday, Wednesday)(Thursday, Thursday) (Friday, Friday) (Saturday, Saturday)

Probability of reaching on same day = 5/25

ii) Number of combinations of consecutive days (Tuesday, Wednesday) (Wednesday, Thursday) (Wednesday, Tuesday) (Wednesday, Thursday) ……………………………hence we will get 8 combination

Probability of reaching on consecutive days= 8/25

Answer 5. Apoorva throws two dice so total outcome = 36

Number of outcomes for getting 36 n(E) = 1 (6×6)

So, probability of getting 36 for Apoorva = 1/36

Pihu throws one dice so total number of outcomes for pihu = 6

Number of outcomes for getting square of number 36 n(E) = 1 (square of 6 = 36)

So probability for getting square of number is 36 = 1/6

Hence Pihu has better chance to get number 36

Answer 6. I) Number of jacks of red suit in the deck = 2

Total number of cards = 52 so probability of getting king = 2/52

ii) Number of diamond card in the deck = 13

So, probability of getting a diamond card= 13/52

iii) Probability for 5 or 9 of club = 2/52

Answer 7. Total possible outcome =36

Probability for getting 3 =	2/36
Probability for getting sum 4 =	3/36
Probability for getting 5 =	4/36
Probability for getting sum 6	5/36
Probability for getting sum 7	6/36
Probability for getting sum 8	5/36
Probability for getting sum 9	4/36
Probability for getting sum 10	3/36
Probability for getting sum 11	2/36
Probability for getting sum 12	1/36

No, I do not agree with the argument as the result shows above.

Answer 8. X + (x/2) +2 =14

(3x/2) = 14-2

X = 8

Probability of getting a black marble = 8/14

Probability of getting a white marble= 3/7

Answer 9 Total number of cards =17

i) Odd numbered cards = 1,3,5,7,9,11,13,15,17

Possible outcomes = 9 so, probability of getting odd numbered card= 9/17

ii) Prime numbered cards= 2,3,5,7,11,13,17 = 7 cards

So, probability of getting prime numbered card = 7/17

iii) Number divisible by 3 = 3,6,9,12,15 i.e. 5

Hence probability of getting card with number divisible by 3 = 5/17

Answer 10: i) cards with an odd number less than 30 are –

7,9,11,13,15,17,19,21,23,25,27,29

Total number of outcomes = 65 (from 6 to 70)

Total favourable outcome = 12

So, probability of a card with odd number = 12/65

ii) Cards with composite number between 50 and 70 are-

51,52,54,55,56,57,58,60,62,63,64,65,66,68,69 (i.e., 15 cards)

Total number of cards = 65

So, probability of getting a composite number = 15/65 i.e., 3/13

Answer 11. 0 ½ 1 2

Let E be event that the music is stopped within the first half minute

The outcome favourable to E are on the number line from 0 to ½

The distance from 0 to 2 is 2 while distance from 0 to ½ is ½

Since all events are equally likely so total distance =2 and

Distance favourable to the event is ½

P (E) = (½)/2

P (E) = 1/4

Answer 12. Let number of green balls be x so number of blue balls will be = 24-x

Probability of getting green ball = x/24

According to question x/24 = 2/3

X = 16

Hence number of blue balls = 24- 16

= 8

Answer 13. Let total number of eyes be = 100

No. of black eyes = 65, no. of brown eyes = 25

Remaining are blue eyes = 100- (65+25) = 10

Probability of getting blue eyes = 10/100 i.e., 1/10

Probability of getting brown or black eyes = 90/100 i.e., 9/10

Probability of getting blue or brown eyes = 75/100 i.e., ¾

Answer 14. Total number of pieces = 8+ 10 =18

i) No. of triangles = 8 hence probability of getting triangle =8/18

= 4/9

ii) No. of squares = 10 hence P(square is lost) = 10/18

= 5/9

iii) no. of triangles of red colour = 8-3 =5

So, P (triangle of red colour is lost) = 5/18

Answer 15. Number of cards removed = 2 red queens + 2 black jacks = 4 cards

Total number of remaining cards = 52- 4 = 48

i) There are 4 kings in remaining cards

Hence probability of getting a king = 4/48 i.e. 1/12

ii) After removing 2 red queens, 24 red cards are left hence

Probability of getting red colour card = 24/48 i.e. 1/2

iii) After removing 2 red queens and 2 black jacks 8 face cards are left hence

Probability of getting a face card = 8/48 i.e. 1/6

iv) Only 2 black queens are left hence

Probability of getting a queen = 2/48 i.e. 1/12

Answer 16. Favourable outcome for value more than 1 rupee= 30

Total number of coins = 180

Probability of falling a coin of value more than 1 = 30/180

i) Favourable outcome for value less than 5 rupees = 170

Probability of falling a coin value less than 5 rupees = 170/180 i.e., 17/18

Answer 17 i) those numbers from 2 to 201 which are perfect square are

4, 9, 16, 25, 36, 49, 64, 81, 100, i.e. 9 cards marked with numbers perfect Square

Favourable outcome = 9

Total outcome = 100 so probability of getting a perfect square number = 9/100

Prime numbers less than 20 are 2,3,5,7,11,13,17 and 19 i.e., 8 cards

Favourable outcome = 8 and total outcome = 100

So, probability of getting card with prime number = 8/100 i.e., 2/25

Answer 18. Total number of outcomes = 5+8+4 = 17

Number of red marbles = 5

Probability of getting red marble = 5/17

Ii) Probability of getting green marble = 4/17

Probability of getting not green marble = 1-(4/17) i.e., 13/17

Answer 19 Total balls = 12 l, let white balls be x

Probability of getting white ball = x/12

If 6 more balls are put into bag now total balls = 18

White balls are = 6+x

ii) Probability of getting white balls = (x+6)/18

According to question (x+6)/18 = 2(x/12) on solving this we get x =3

Answer 20. Multiple of 6 first 200 integers: 6, 12, 18, 24, 30, 36, 42, 48………………… 198

Multiple of 8 first 200 integers: 8, 16, 24, 32, 40, 48, 56………………………… 200

Number of multiple of 6 or 8 = 50

Probability of getting integer multiple of 6 or 8= 50/200 = 1/4